Bureau of Supplies and Accounts
U.S. Navy

DYNAMIC MODELING OF INVENTORIES SUBJECT TO OBSOLESCENCE

June 1962

C-E-I-R INC.
LOS ANGELES CENTER:
9171 Wilshire Boulevard, Beverly Hills, California

DYNAMIC MODELING OF INVENTORIES SUBJECT TO OBSOLESCENCE

A Report of Research Performed For
The Office of Advanced Logistics Research
Bureau of Supplies and Accounts
U.S. Navy

June 14, 1962

Contract No: Nonr 3333(00)

C-E-I-R, Inc.
Los Angeles Center
9171 Wilshire Boulevard
Beverly Hills, California

ACKNOWLEDGEMENTS

The research reported in this document was carried on by John Y. Lu and Robert J. Wolfson of the C-E-I-R, Los Angeles Center and the report was written by them. The Project Director was Robert J. Wolfson. Significant contributions were made by Lester R. Ford, Jr., and George W. Brown of C-E-I-R, Los Angeles, as consultants. Ford was responsible for the mathematical development of the model reported in Chapter IV. Brown was responsible for the mathematical development of the models reported in Chapter VI. In the early stages of the work, David N. Walker and J.C. Miller, also of C-E-I-R, Los Angeles, provided valuable aid.

In addition, we are most grateful to Mr. J.R. Simpson, Manager of Advanced Logistics Research, Bureau of Supplies and Accounts, U.S. Navy, for his valuable advice and his patience and forbearance. Cdr. Dale W. Kesselring, and Mr. C.R. Fickes, of the Office of Advanced Planning Research, Aviation Supply Office, U.S. Navy were most helpful in familiarizing us with the operations of the ASO and in providing access to ASO records. It is needless to add that any errors of fact, logic or taste are due to us.

J.Y.L. and R.J.W.

SUMMARY

This research report describes studies done by C-E-I-R, Inc., under Nonr 3333(00), which are concerned with the impact of obsolescence on inventories of Naval supplies. Obsolescence is examined with the objective in mind of defining it operationally. Several types of dynamic models are developed, making use of the operationally defined motion. Some of these models employ Bayesian procedures for assessment of demand rates. A statistical analysis of some Navy supply data, which was carried out in an attempt to cover empirical relationships between obsolescence and other factors, is described. Finally some changes in inventory control procedures are suggested.

TABLE OF CONTENTS

CHAPTER I: INTRODUCTION

In January 1961, C-E-I-R, Inc., was authorized by the Bureau of Supplies and Accounts to undertake a study of the economic impact of obsolescence on inventory costs and control. This document is a report of results.

The study has led to the development of a sequence of models of inventory costs, computing techniques associated with these models, and a proposal for installation of inventory control procedures based on the models described herein. The models deal with inventory operation costs under a variety of assumptions as to: lead time, inventory carrying charges, ordering costs, disposal costs, shortage costs, probability distribution of demand for an item, probability of incidence of obsolescence during any time period and linkage between demands for two or more items.

These models have one feature in common: all are dynamic models. That is, a steady state is not assumed; the possibility exists for variation of demand and obsolescence distributions, as well as all the costs of operation of the inventory. Several of the more sophisticated models incorporate Bayesian features. Taken together, these two features, the dynamic program and the Bayesian features, lead to an ability to adapt in a continuous fashion to changes both in the structure of the environment and in a priori statements whose validity is revealed only slowly as experience is generated.

Central to this study is an attempt to understand and model the nature and impact of obsolescence of items in inventory. The following chapter is concerned with a discussion of what is meant by obsolescence, as well as some concern with the magnitude of obsolescence of inventories of Naval Supplies.

CHAPTER II: OBSOLESCENCE: DEFINITION AND IMPORTANCE

It has been standard practice in the U.S. Navy supply system to deal with obsolescence as a blanket charge to be assessed equally across all Navy Supply items. The rate at which this charge has customarily been assessed is 10% of the total value of the inventory per year.

The total value of inventories of Naval Aircraft spare parts and supplies held by the Navy amounts to approximately $2.5 billion. Clearly, the total value of all inventories held by the Navy is much larger than this amount. Consequently, the total obsolescence charges under the present system amount to several hundred million dollars per year.

It seems indisputable that better understanding of the specific impact of obsolescence, and the development of inventory control procedures which take the obsolescence process into account in a realistic fashion, should lead to considerable economies in the use of Navy Supply budgets.

Defining the term <u>obsolescence</u> is not quite as simple, in the context of a study of the sort under discussion, as it would be for the purposes of ordinary discourse. Something is <u>obsolescent</u>, according to Webster's New Collegiate Dictionary (1960 Edition), when it is going out of use. In ordinary discourse, it is assumed that there is no serious problem involved in ascertaining whether or not something is going out of use. However, for our purposes, it turns out that this is a most difficult matter to settle until after the fact.

Establishing that something is <u>obsolete</u>, that is, establishing that something has already gone out of use; is a much simpler problem.

In this case, there are several circumstances which might have led to such a situation.

1. An item has gone out of use because the function served by that item is no longer required; i.e., demand has disappeared.
2. An item is going out of demand because whenever a unit of the item wears out, breaks, or is consumed, the item is replaced by a different item which performs similar or identical functions; i.e., the item is going out of use.
3. An item has gone out of demand because, regardless of consumption, wearout or breakage, as soon as replacements are available, all units of that item currently in use are replaced by units of other items which serve similar or identical functions, that is, the item has gone out of use.

So far as the supply system is concerned, being out of use, and not being in demand are the same condition.

<u>Once something has become obsolete, the fact is clear</u>. But there is not much which can be done at such a time to face the fact, beyond clearing out and disposing of any remaining inventories of such items.

something is going out of use. However, for our purposes, it turns out that this is a most difficult matter to settle until after the fact.

Establishing that something is <u>obsolete</u>, that is, establishing that something has already gone out of use; is a much simpler problem.

In this case, there are several circumstances which might have led to such a situation.

1. An item has gone out of use because the function served by that item is no longer required; i.e., demand has disappeared.
2. An item is going out of demand because whenever a unit of the item wears out, breaks, or is consumed, the item is replaced by a different item which performs similar or identical functions; i.e., the item is going out of use.
3. An item has gone out of demand because, regardless of consumption, wearout or breakage, as soon as replacements are available, all units of that item currently in use are replaced by units of other items which serve similar or identical functions, that is, the item has gone out of use.

So far as the supply system is concerned, being out of use, and not being in demand are the same condition.

<u>Once something has become obsolete, the fact is clear</u>. But there is not much which can be done at such a time to face the fact, beyond clearing out and disposing of any remaining inventories of such items.

However, if it were possible to detect accurately the process of obsolescence in its earlier stages, while it has not run its course, then there might be some steps which might profitably be taken by an inventory manager. Consider, for example, the following very simple case:

Let:

Carrying Cost = h

Stockout Cost = π

Liquidation Cost = k

Demand = D

Probability distribution of demand = $p(D)$

$$\sum_{D=0}^{r} p(D) = P(r) = \text{Prob } (D \leq r)$$

A. It is not known that obsolescence will occur at the end of this period. An optimal policy is followed, then obsolescence occurs and stock is liquidated.

The expected inventory costs the manager can plan on are:

$$(2.1) \quad C(I_o) = h \sum_{D=0}^{I_o} (I_o - D)\, p(D) + \pi \sum_{D=I_o+1}^{\infty} (D - I_o)\, p\,(D)$$

If I_o is optimal

$$(2.2) \quad C(I_o + 1) - C(I_o) \geq 0$$

$$(2.3) \quad C(I_o - 1) - C(I_o) \geq 0$$

(2.4) $$C(I_o+1) = h\sum_{D=0}^{I_o} (I_o-D)\ p(D) + h\sum_{D=0}^{I_o} p(D) + \pi \sum_{D=I_o+1}^{\infty} (D-I_o)\ p(D)$$

$$- \pi \sum_{D=I_o+1}^{\infty} p(D)$$

(2.5) $$C(I_o+1) - C(I_o) = P(I_o)\ (h+\pi) - \pi \geq 0$$

Similarly,

(2.6) $$-\left[C(I_o-1) - C(I_o)\right] = P\ (I_o-1)\ (h+\pi) - \pi \leq 0$$

(2.7) $$P(I_o-1) \leq \frac{\pi}{h+\pi} \leq P(I_o)$$

B. It is known that obsolescence will occur at the end of the period. The manager plans accordingly. His expected costs are:

(2.8) $$E(I_*) = h\sum_{D=0}^{I_*} (I_*-D)\ p(D) + \pi \sum_{D=I_*+1}^{\infty} (D-I_*)\ p(D) + k\sum_{D=0}^{I_*} (I_*-D)\ p(D)$$

(2.9) $$E(I_*) = (h+k) \sum_{D=0}^{I_*} (I_*-D)\ p(D) + \pi \sum_{D=I_*+1}^{\infty} (D-I_*)\ p(D)$$

By the same reasoning as (2.1) to (2.7)

(2.10) $$P(I_*-1) \leq \frac{\pi}{\pi + h + k} \leq P\ (I_*) \qquad \text{so } I_* \leq I_o$$

The total expected costs incurred in situation A. must include liquidation of remaining stock:

(2.11) $$E(I_o) = C(I_o) + k \sum_{D=0}^{I} (I_o-D)\ p(D) = (h+k) \sum_{D=0}^{I} (I_o-D)\ p(D)$$

$$+ \pi \sum_{D=I_*+1}^{\infty} (D-I_*)\ p(D)$$

$$(2.12)\quad E(I_o) - E(I_*) = (h+k)\left[\sum_{D=0}^{I_o} (I_o-D)\ p(D) - \sum_{D=0}^{I_*} (I_*-D)\ p\ (D)\right]$$

$$+\pi\left[\sum_{D=I_o+1}^{\infty} (D-I_o)p(D) - \sum_{D=I_*+1}^{\infty}(D-I_*)\ p(D)\right]$$

It is easily shown (see Footnote 1) that (2.12) is positive.

---

[1] Starting with (2.12): adding $(h+k)\left[\sum_{D=I_o+1}^{\infty}(I_o-D)\ p(D) - \sum_{D=I_*+1}^{\infty}(I_*-D)p(D)\right]$

$$-\ (h+k)\left[\sum_{D=I_o+1}^{\infty} (I_o-D)\ p(D) - \sum_{D=I_*+1}^{\infty} (I_*-D)\ p(D)\right]$$

we get

$$(2.13)\quad E(I_o)-E(I_*)=(h+k)\left[\sum_{D=0}^{\infty} (I_o-D)\ p(D) - \sum_{D=0}^{\infty} (I_*-D)\ p(D)\right]$$

$$+\ (h+k+\pi)\left[\sum_{D=I_o+1}^{\infty}(D-I_o)\ p(D) - \sum_{D=I_*+1}^{\infty}(D-I_*)\ p(D)\right]$$

$$(2.14)\quad E(I_o)-E(I_*)=(h+k)(I_o-I_*)+(h+k+\pi)\left[\sum_{D=I_o+1}^{\infty} (D-I_o)\ p(D)\right.$$

$$\left.-\sum_{D=I_o+1}^{\infty} (D-I_*)\ p(D) - \sum_{D=I_*+1}^{I_o} (D-I_*)\ p(D)\right]$$

$$(2.15)\quad E(I_o)-E(I_*)=(h+k)(I_o-I_*)+(h+k+\pi)\left[\sum_{D=I_o+1}^{\infty} (I_*-I_o)\ p(D)\right.$$

$$\left.-\sum_{D=I_*+1}^{I_o} (D-I_*)\ p(D)\right]$$

$$(2.16)\quad E(I_o)-E(I_*)=(h+k)(I_o-I_*)+(h+k+\pi)\left\{(I_*-I_o)\ \left[1-P(I_o)\right]\right.$$

$$\left.-\sum_{D=I_*+1}^{I_o} (D-I_*)\ p(D)\right\}$$

$$(2.17)\quad E(I_o)-E(I_*) = (h+k)(I_o-I_*) + (h+k+\pi)$$

$$\left\{(I_*-I_o)-(I_*-I_o)\left[P(I_*) + \sum_{D=I_*+1}^{I_o} p(D)\right] - \sum_{D=I_*+1}^{I_o} (D-I_*)p(D)\right\}$$

$$(2.18)\quad E(I_o)-E(I_*) = -\pi(I_o-I_*) + (h+k+\pi)(I_o-I_*)\ P\ (I_*)$$

$$+\ (h+k+\pi) \sum_{D=I_*+1}^{I_o} (I_o-I_* -D+I_*)\ p(D)$$

$$(2.19)\quad E(I_o)-E(I_*) = -\pi\ (I_o-I_*) + (h+k+\pi)(I_o-I_*)\left(\geq \frac{\pi}{h+k+\pi}\right)$$

$$+\ (h+k+\pi) \sum_{D=I_*+1}^{I_o} (I_o-D)\ p(D)$$

$$(2.20)\quad E(I_o)-E(I_*) = \underbrace{-\pi\ (I_o-I_*) + (\geq \pi(I_o-I_*))}_{>0} + \underbrace{(h+k+\pi)\sum_{D=I_*+1}^{I_o}(I_o-D)p(D)}_{>0}$$

We have made use of the fact that

$$P(I_*) \geq \frac{\pi}{\pi+h+k} \quad \text{and} \quad I_* < I_o$$

Th: If $I > J$ and $P(J) > \frac{\pi}{\pi+h+k}$

then $E(I) > E(J)$

We now consider a numerical example:

$h = 60$

$\pi = 500$

$k = 200$

Demand is Poisson distributed, with $\lambda = 6$, $p(D) = \frac{e^{-6}(6)^D}{D!}$

$$\frac{\pi}{\pi+h} = \frac{500}{560} = .8928 \quad I_o = 9$$

$$\frac{\pi}{\pi+h+k} = \frac{500}{860} = .5814 \quad I_* = 6$$

from (2.18)

$$(2.18') \quad E(I_o)-E(I_*) = -500(3) + (860)(3)(.6063) + (806) \sum_{D=7}^{9} (9-D)\ p(D)$$

$$= -1500 + (2580)(.6063) + 806 \quad 2 \times .1377 + 1 \times .1033$$

$$= 369.4862$$

Total costs under situation B are 732.5640

Thus, there is clear gain of the order of 50% from having had prior knowledge of the impact of obsolescence.

In cases where items go out of demand or out of use as a consequence of administrative decisions, it might be easier to anticipate the process of obsolescence than in other cases. This is so because administrators must have some reasons for making such decisions, and these reasons are probably open to examination. However, in cases where administrative decisions are not involved, it is necessary for inferences to be made about the future state of demand from the present state of demand.

Such inferences are of the form: given that demand in the present period is equal to y, the probability that demand M months from now will be less than, or equal to $\bar{y}$ is $P(\bar{y};y,M)$. When P is sufficiently high, it may turn out that the cost of maintaining the item in inventory for demand levels as low as, or lower than, $\bar{y}$ is more than the expected costs which would be incurred if the system were out of stock of the item.

In such a circumstance a declaration that the item is to be treated as if it were obsolescent might be in order. This would mean taking some action which would lead to one of the three conditions 1), 2), or 3), which are described above, (on page 4).

In the models which are developed and discussed in the following sections of this report there are, basically, two approaches taken to the notion of obsolescence.

The first, a simpler treatment, is concerned with the development of dynamic disposal-procurement policies under the assumption of a probability distribution of time of incidence of obsolescence which is known a priori, and which is not changed systematically within the model.

This model is very flexible so far as input data are concerned; any form of obsolescence probability distribution, demand distribution, or inventory costs may be used. Consequently, as ancillary information about any of these is developed, new assumptions can be introduced into the model.

Numerical analysis of this model has been carried out fairly extensively. The results of the numerical analysis tend to support conjectures as to the extensibility and greater generality of some theorems which have appeared in the literature recently. Moreover, this model appears promising as the basis for a first step toward the rationalization of procurement-disposal stock control of Naval inventories.

The second approach is a much more original treatment. Indeed, there are no results in the literature known to us which incorporate features to be found in this second group of models. In this approach, which also involves dynamic models, there is assumed a priori the form of the probability distribution of the time of incidence of obsolescence, and the parameter values for this distribution. However, the parameter values are modified, within the model, by Bayesian procedures involving a functional relationship between the recent history of demand for the line item and the parameter values of the obsolescence distribution.

This basic approach is adapted in several ways in the models of this sort which are discussed herein. In one model the crucial effect of demand history appears when demand is at a level of zero. In another, there is incorporated a Markov process which deals with the transition between various levels of demand. In a third, there is added on a feature involving the dependency of demand for one item upon the demand for another related (complementary or substitute) item.

What is true of all these models is their dynamic character. They are true dynamic programs which optimize with respect to a cost function which includes all cost elements of an inventory system except relocation of existent inventory within the system, and repair of items. Thus, this family of models does not deal with questions of how, within an inventory system, the inventory should be dispersed. Neither does it deal with repair vs. purchase, nor with level-of-complexity of items (the problem referred to by A.J. Clark as the "Inventory Range Problem.")

In subsequent chapters these models will be dealt with in detail. In addition, there will be some discussion of the results of statistical analysis of data on some 4000 aircraft spares which have been declared obsolete during the past year. Finally, there will be some discussion of explorations which were undertaken in connection with the development of the most sophisticated of the inventory models which use the properties of a class of probability distributions that reproduce under the operation of taking the conditional distribution. The use of elements of this general class of functions in these models to represent the probability distribution of time to obsolescence leads to great simplification of the computing problem. This is so because for these functions the parameters are computable by simple recursions which are time-independent.

CHAPTER III

SURVEY OF INVENTORY MODELS WITH EXPLICIT TREATMENT OF OBSOLESCENCE

Numerous investigations have been made of the problems of inventory control in the past decade. Although most of them have dealt with the determination of optimum policies with which to regulate inventories in the absence of possible obsolescence of the inventories, there are still a number of studies in which some aspect of the problem of obsolescence was explicitly treated. This chapter is essentially an annotated bibliography of the studies on inventory obsolescence. The bibliography is by no means exhaustive, but every study which, in our opinion, made significant contributions to this area of inventory control study is included.

Results which emerged from these inventory obsolescence studies are varied depending on: (1) the assumption regarding the occurrence of obsolescence; (2) the nature of the problem that the investigator has to deal with; and (3) the approach adopted for obtaining an optimal solution. In order to bring some order to these widely disparate results and facilitate further discussion, an attempt has been made to arrange the material to be discussed in a systematic way in which the organization is based on the problem-oriented viewpoint rather than on the technique-oriented viewpoint. The criteria for classification are:

1. Deterministic vs. probabilistic problem. If the parameter which specifies the occurrence of obsolescence, such as the

length of time-to-obsolescence, is known with certainty, such a problem is referred to as deterministic. If it is known in terms of a probability distribution, it is called the probabilistic problem. A problem will be also considered "deterministic" if obsolescence enters the problem only as a non-stochastic function of any inventory parameter.

2. The objectives of the models. Inventory problems can be classified according to the objective that the investigator sets to accomplish. The following three distinct objectives are noted in the studies to be reviewed here:

   a. The main objective is to determine the optimum economic lot size for a single item taking cognizance of obsolescence.

   b. The determination of the optimum inventory for a single item if no additional procurements are to be made. This optimum inventory is commonly referred to as final inventory.

   c. To determine the (s,S) type ordering and disposal policies taking account of obsolescence cost.

In Table I, the inventory obsolescence studies are classified according to the criteria discussed above and they will be discussed at some length in the order indicated in each entry of the table.

Table I: Classification of Inventory Obsolescence Studies

OBJECTIVE	Assumption about Occurrence of Obsolescence: Deterministic	Assumption about Occurrence of Obsolescence: Probabilistic
Determination of EOQ	**Case 1** Whitin [13]	**Case 2** Grassi and Gradwohl [6]
Determination of final inventory	**Case 3** Hadley and Whitin [7]	**Case 4** Hadley [8] Mohan and Garg [11] Simpson [12]
Determination of (s,S) type ordering and disposal rules	**Case 5** Barankin [2] Fukuda [5]	**Case 6** Allen and Broida [1] Barankin and Denny [2] Ford [4] Brown [3]

Case 1: Deterministic EOQ Model

The solution of the economic order quantity problem, according to Whitin [13], has a rather long history. It is, perhaps, still one of the most frequently used inventory formulae. In the formula for determining economic order quantities, obsolescence cost is treated as a constant percentage of inventory carrying cost which increases as inventories increase. This carrying cost is to be balanced against those cost items such as quantity discounts, freight differentials which decrease as inventories increase. The derivation of the formula is as follows:

Let

Y = expected demand per period

Q = economic order quantity

C = unit inventory carrying cost per period. (This is made up of material, interest, depreciation and obsolescence costs)

S = procurement expense per order

Then total variable cost (TVC) per period is

$$TVC = \frac{QC}{2} + \frac{Y}{Q} S \tag{3.1}$$

Upon differentiating (1) with respect to Q and setting the resulting derivative equal zero, we obtain the minimizing value of Q by solving the derivative for Q.

$$(3.2) \qquad Q = \sqrt{\frac{2YS}{C}}$$

As can be seen in the above formula, if the risk of obsolescence should increase, this will be reflected in a higher inventory carrying cost and, consequently, a smaller economic order quantity.

In addition to the EOQ model described above, many traditional inventory models treat obsolescence cost as a part of carrying cost. However, when obsolescence becomes a major cost in an inventory model, a more explicit determination of obsolescence costs becomes desirable.

Case 2. Probabilistic EOQ Model

In this model obsolescence cost is formed as a function of the obsolescence rate by assuming that the probability of obsolescence at a future time can be expressed by a certain distribution function. Grassi and Gradwohl [6] have obtained a probabilistic EOQ formula by assuming the life span of an item to follow an exponential density function, $f(t) = \mu e^{-\mu e}$, where μ is the obsolescence or death rate.

Their model may be stated as follows:

Let

P = unit order cost

D = the sum of unit material, labor and overhead costs

S = setup cost per order

R = expected demand per period

B = the safety or buffer stock

I = unit inventory holding cost per period

β = the inventory charge rate per period

E = expected unit obsolescence cost

Then the sum of all the costs associated with ordering one unit of inventories is:

$$C = P + I + E$$

Each component of the total unit cost C is given by:

$$P = D + \frac{S}{Q} \tag{3.3}$$

P may be regarded as the value of one unit of inventories.

$$I = \left(B + \frac{Q}{2}\right)\left(D + \frac{S}{Q}\right)\frac{\beta}{R} \tag{3.4}$$

$B + \frac{Q}{2}$ is the average inventory level under the assumption that a quantity Q will be ordered when the inventory level reaches the safety level B; the product $(B + \frac{Q}{2})$ $(D + \frac{S}{Q})$ is the value of the average inventory stock. The inventory charge is made on this average inventory stock and is prorated over the total expected requirement for one period.

The expected unit obsolescence cost may be calculated based on the following consideration: Suppose a lot size of Q is ordered for an item when the item is at age t years (unit for measuring the passage of time is arbitrary). The lot will be used up when the item is of age $T + \frac{Q}{R}$ years. It is then of interest to know the probability of obsolescence during the time interval $\frac{Q}{R}$, given non-obsolescence prior to t so that the expected loss due to obsolescence during the time interval of length $\frac{Q}{R}$ can be calculated.

Because of the assumption that the life span of the item follows an exponential distribution, the probability that obsolescence occurs prior to t (or, alternatively, the probability that the item's life span is, at most, as long as t) can be stated as follows:

$$\Pr(T \leq t) = F(t) = 1-e^{-\mu t}$$

where T represents the age at obsolescence of that particular item in question.

The conditional probability $F(\tau | t_o)$ of obsolescence in an additional time τ after experiencing non-obsolescence to t_o is given by

$$G(\tau) = \frac{F(t_o + \tau) - F(t_o)}{1-F(t_o)} = 1-e^{-\mu\tau}$$

Hence the conditional density function is

$$\frac{dG(\tau)}{d\tau} = g(\tau) = \mu e^{-\mu\tau}$$

The expected number of unit lost due to obsolescence during the time interval t to $t + \frac{Q}{R}$ is:

$$= \int_0^{\frac{Q}{R}} (B + Q - R\tau) \; g(\tau) \; d\tau$$

(3.5) $$= (B-\frac{R}{\mu}) \; (1-e^{-\mu(Q/R)}) + Q$$

The expected unit loss due to obsolescence becomes

$$(3.6) \qquad E = \left(D + \frac{S}{Q}\right) \frac{(B-R/\mu)\left(1-e^{-\mu(Q/R)}\right)}{Q} + 1$$

The optimum economic order quantity is obtained by finding the minimizing value of Q in the following expression

$$C = P + I + E$$

$$(3.7) \qquad = \left(D + \frac{S}{Q}\right)\left[2 + \left(B + \frac{Q}{2}\right)\frac{\beta}{R} + \frac{(B-R/\mu)\left(1-e^{-\mu(Q/R)}\right)}{Q}\right]$$

The resulting solution given by Grassi and Gradwohl is

$$(3.8) \qquad Q = \sqrt{\frac{2RS\left[1 + \frac{B}{R}(\mu + \beta)\right]}{D\left[(\mu + \beta) - \frac{B\mu^2}{R}\right]}}$$

It is interesting to note that if the conditional density $g(\tau)$ follows a rectangular distribution, i.e., obsolescence is equally likely to occur at any moment after the item has survived up to t, the new EOQ formula is exactly the same as (3.8) except the second term in the denominator $\frac{B\mu^2}{R}$ vanishes.

In using the formula (3.8) the level of buffer stock β is assumed to be determined outside of the model. If the joint determination of Q and B is desired, it may be achieved by appending penalty cost due to stock shortage to the cost function (3.7) and minimizing the function with respect to B and Q.

Case 3. Final Inventory Model with a Known Obsolescence Date

The problem is to determine the optimal amount of inventory on hand if no additional procurements are allowed, and, furthermore, a date of obsolescence is known with certainty. This inventory is commonly known as the optimum final inventory. The main feature of such a problem is that it involves two types of cost; one is incurred at a fixed point in time (obsolescence cost) and another is a function of time (holding and stockout costs).

Hadley and Whitin [7] formulated a model which deals with problems such as above and obtained a solution assuming the demand for the item to be Poisson with the mean demand rate independent of time

Let k = the liquidation loss per unit at the time of obsolescence

h = the carrying cost per unit per unit time

π = the stockout cost per unit per unit time

$p(X; \lambda t)$ = the Poisson probability distribution that the demand is exactly X units in a time period of length t with λ being the average demand rate

T = time of obsolescence

H = the optimal final inventory

The expected cost of holding H units from time o to T is the sum of the expected costs of disposing the unused amount at T; carrying inventory, and stockouts for the time interval o to T. These cost components may be stated

The expected disposal cost:

$$K \sum_{X=0}^{H-1} (H - X)\ p(X ; \lambda T) \tag{3.9}$$

The expected carrying cost:

Since the carrying cost "rate" at time t is

$$h \sum_{X=0}^{H-1} (H - X)\ p(X : \lambda t)$$

the total carrying cost for the interval 0 to T is

$$h \int_0^T \sum_{X=0}^{H-1} (H - X)\ p(X : \lambda t)\ dt \tag{3.10}$$

the expected stockout cost:

Similarly, the total stockout cost for the interval 0 to T is

$$\pi \int_0^T \sum_{X=H}^{\infty} (X - H)\ p(X : \lambda T)\ dt \tag{3.11}$$

The expected cost of holding H which is the sum of (3.9), (3.10) and (3.11) can be minimized by noting the identity

$$\int_0^T p(X ; \lambda t)\ dt = \frac{1}{\lambda} \sum_{u = X+1}^{\infty} p(u, \lambda T)$$

This model is neat and simple to apply. However, its usefulness may be limited for the following reasons:

1. In general, it is not advisable to assume that an obsolescence date can be specified with certainty.

2. In the lifetime of an item, the practice of repeated procurements may be more common than the situation represented by the model.

Case 4: Final Inventory Models with Probabilistic Obsolescence Date

One of the objections to the model in Case 3, is in regard to the assumption of known obsolescence date. This assumption was relaxed in a more recent model developed by Hadley, [8].

He assumed that the date of obsolescence can be described by a continuous density function $r(T)$. Then the probability that the item will become obsolete in the interval T to $T + dT$ is $r(T)dT$. With this new assumption regarding T, the expected cost of holding H may be modified by taking the expected value of (3.9), (3.10), and (3.11) with respect to T.

$$E(H) = k \sum_{X=0}^{H-1} (H - X) \int_0^{\infty} r(T)\, p(X; \lambda T)\, dT + h \int_{T=0}^{\infty} \int_{t=0}^{T} \sum_{X=0}^{H-1} (H - X)\, p(X; \lambda t)\, r(T)\, dT\, dt + \pi \int_{T=0}^{\infty} \int_{t=0}^{T} \sum_{X=H}^{\infty} (X - H)\, p(X; \lambda t)\, r(T)\, dT\, dt \tag{3.12}$$

Using the technique described in the earlier paper by Hadley and Whitin [7], the cost function (3.12) can be minimized. Hadley [8] has also developed another model in which only a finite number of possible dates of obsolescence is assumed and the probability of each is specified.

Another type of final inventory model was developed by Simpson [12] and Mohan and Garg [11]. Their model is distinguished from Hadley's model in that only a knowledge of the average annual demand is assumed, and it will first calculate the optimal economic retention period from consideration of the balancing of carrying charges and the cost of disposal. More specifically, the cost of retaining a unit value of the stock is weighed against the cost of disposing one unit value of the stock now and procure it again at some later stage. It is clear that units will be added to the retention stock as long as the latter cost exceeds the former. On the other hand, the retention stock will be reduced if the former costs exceed the latter. The equilibrium is reached when the cost of storing the marginal unit is exactly equal to the cost incurred for not storing that unit.

Their model may be stated as follows:

Let U = average annual demand

x = the number of years for which a stock, say N, will meet average demand. Note the relation $xU=N$

D = fraction of unit value of material which will be realized in disposal sales

i = interest rate

r = annual storage cost of material, expressed as a fraction of the unit value of material

C_R = total cost of retaining the unit value of stock

C_D = total cost of disposing the unit value of stock

F_t = the probability that the item becomes obsolete prior to the (t+1)th year

The retention cost C_R consists of two types of costs, (1) the storage cost for x years, (2) the obsolescence cost.

The first type of cost may be stated as

$$r \sum_{t=1}^{x} (1 - F_t)(1 + i)^{x-t+1}$$

$(1 - F_t)$ is the probability of non-obsolescence before the (t+1)th year, and $(1 + i)^{x-t+1}$ is the compound interest charge for storing the unit value of stock for $x-t+1$ years.

The obsolescence cost is the product of the unit value of stock and the probability of obsolescence F_x.

Hence

$$C_R = F_x + r \sum_{t=1}^{x} (1 - F_t)(1 + i)^{x-t+1} \tag{3.13}$$

For each unit value of stock disposed now, the cost equivalent to the unit value will be incurred in order to procure it again after x years assuming there is no price change in the future. However, for each unit value of stock disposed, a salvage value of D is acquired. This amount should increase at compound interests for x years and should be considered as a credit. Therefore, the disposal cost C_D is

$$C_D = 1 - D(1 + i)^x \tag{3.14}$$

The optimum retention period x is found by solving the equation $C_R - C_D = 0$.

The only difference between the result obtained by Simpson and that obtained by Mohan, et al is that the former assumes that the probability of obsolescence follows a uniform distribution while the latter uses a normal distribution.

Simpson gives a detailed account of the advantages and the shortcomings of this retention and disposal formula from the practical point of view. His conclusion is that despite the fact that many rigid assumptions are required to derive this formula, it provides a workable, practical decision rule.

Case 5: Optimal Ordering and Disposal Policies with Known Obsolescence Date

When the items in inventory are known to become obsolete in the future, it is advisable to consider the disposal of surplus items as well as the procurement of stock to meet the future demand.

Consider the following situation in which an item will no longer be used after a certain date and the stock on hand on that date will have to be disposed. Meanwhile, prior to the occurrence of obsolescence, the stock level of this item is reviewed periodically, say, at the beginning of a finite number of equal time intervals; and one of the following decisions is made

1. Procurement of additional stock
2. Disposal of excess stock
3. No procurement and no disposal

For such a multi-stage decision process, the technique of dynamic programming has been known to furnish a suitable framework for analysis. In the following the recursion relation of a dynamic programming model appropriate to the situation described above is stated.

Let us introduce the following definitions:

s = demand per period

x = "initial" inventory level; an inventory level at the beginning of a period before the order or disposal is made

y = "starting" inventory level; an inventory level at the beginning of a period immediately after the order or disposal is made

$r(z)$ = the cost of ordering z units

$d(z)$ = the cost of disposing z units. If the disposal item has a salvage value, this cost assumes a negative value.

$h(z)$ = the cost of holding z units of inventory for one period

$p(z)$ = the cost per period of having demand z units greater than inventory

ρ = a discount factor

It is assumed that there is no delay either in the order or in disposing of stock. Suppose there are n periods before obsolescence occurs. The policy to be considered is of the following simple forms: for each period a starting inventory y is specified for each value of initial inventory x. Associated with each y is a certain expected cost. The problem is to minimize this expected cost by suitable choice of y.

Let us define

$C_k\ (x)$ = expected total cost for a process which has k periods remaining and starting the process with x units of initial inventory with an optimal ordering and disposal policy.

Since this total expected cost is composed of: (1) a cost of using an optimal policy in this period, (2) a discounted future cost, we have the following recursion relation

$$(3.15)\quad C_k(x) = \underset{y\geq 0}{\mathrm{Min}} \left\{ r(y-x) + d(x-y) + E\left[h(y-s) + p(s-y)\right] + \rho\ E\ C_{k-1}(y-s) \right\} \quad \text{for} \quad k = 2,3,\ldots,n.$$

where the ordering cost and disposal cost functions are zero for negative arguments. E denotes expectation and the expectation operation is with respect to demand s.

When there is only one period remaining in the process, the total expected cost is simplified to the following:

$$(3.16)\qquad C_1(x) = \underset{y\geq 0}{\mathrm{Min}} \left\{ r(y-x) + d(x-y) + E\left[h(y-s) + p\ (s-y)\right] \right\}$$

Together, (3.15) and (3.16) enable us to determine, successively for $k=1,2,\ldots,n$, the optimal value of y for each x and the corresponding minimum expected cost $C_k(x)$.

This is the type of model formulated by Barankin and Denny [2] and Fukuda [5]. Although the cost functions as well as the demand per period are considered to be independent of time in the above model, the same technique still applies even if they should become time dependent. The assumption of no time lag in delivery can be also relaxed and the same technique can be used as long as a time lag is known with certainty.[1]

Case 6: Optimal Ordering and Disposal Policies with Probabilistic Obsolescence Date

Barankin and Denny [2] and Ford [4] have extended the model described in Case 5 to the situation in which the information about obsolescence is available in the form of a probability statement, e.g., the probability of obsolescence in a given period, say the kth period, is d_k. Presumably, $\Sigma d_k = 1$. Then the conditional probability a_k that obsolescence occurs in period k, given non-obsolescence in periods $n, n-1, \ldots, k+1$, may be calculated as follows:

$$a_k = \frac{d_k}{\sum_{i=1}^{k} d_i}$$

With a knowledge of these conditional probabilities for $k=1,2,\ldots,n$ only a simple modification of the recursion relation (3.15) is needed to give us a desired inventory model.

$$C_k(x) = \underset{y \geq 0}{\text{Min}} \left\{ c(y-x) + E\,P\,(s-y) + E\,h(y-s) + a_k d(x-y) + (1-a_k)\,\rho\,E\,C_{k+1}(y-s) \right\} \qquad \text{for } k = 2,3,\ldots,n$$

[1] See H. Scarf, "(s,S) Policy in Dynamic Inventory Problem," Mathematical Methods in the Social Science, Stanford University Press, Standord, California.

Since the conditional probability of obsolescence in the last period a_1 is equal to unity, the total expected cost function for $k=1$, is the same as (3.16). Again, starting with $k=1$, we can successively calculate a set of optimal policies and corresponding expected total costs for each period.

A more detailed discussion regarding the modeling of this type of inventory problem as well as some characteristics of optimum policies is presented in Chapter IV of this report.

In all the dynamic inventory models discussed so far, recursion relation such as (3.15) is always characterized completely by two state variables, x, the stock on hand, and n, the number of remaining periods. Brown [3] considered a more realistic situation in which the latter state describing variable is given in terms of a probability distribution. This introduction of a stochastic variable for describing the nature of process does not unduly complicate the calculations. Results of this type of dynamic inventory model can be found in Chapter VI of this report.

In the study made by Stanford Research Institute for BuSandA, Allen and Broida [1] considered the problem of minimizing the unit-weeks of system-wide shortages subject to the total variable procurement budget (converted into the shortage equivalent) by suitable choice of non-negative order quantities. Since the stock level of each item in the system is reviewed every quarter and an order is placed if necessary to raise the stock level to a certain level which is most desirable from the standpoint of minimizing the shortage risk, Allen and Broida set out to determine what should be considered the most desirable level with a fixed amount of resources available to the system.

The suggested procurement rule is as follows:

Let R = the most desirable inventory level (determined by the model)

I = the sum of stock on order and on hand

(1) If $R - I \leq 0$, no order is placed

(2) If $R - I > 0$, order the difference (R-I) or the economic order quantity--whichever is greater

This procedure may be considered a variant of the usual (s,S) policy. The most interesting feature of their study is that a budget constraint was explicitly taken into account in deriving an optimal inventory regulating procedure.

Since the derivation of their procurement rule is rather lengthy, it will not be reproduced here except to indicate how obsolescence costs are incorporated into their model.

They assumed that the probability of obsolescence at a future time to be expressed by an exponential distribution

$$1-e^{-\gamma t}$$

where γ is the obsolescence factor for the item in question, the probability of non-obsolescence is then $e^{-\gamma t}$. The probability of non-obsolescence at time t is multiplied by the expected number of units short at that time. This yields the expected number of units short at time t when no obsolescence occurs. From this last quantity, the expected number of unit-weeks of shortage is obtained by integrating out t. Similarly, in estimating the budget expenditure needed for placing an order of a certain size, it is weighed by the probability of non-obsolescence.

When obsolescence is treated in the manner described above, it has a net effect of reducing the amount of budget expenditure available for procurement purposes; at the same time it lowers the risk of shortage because when the item has already become obsolete there is no question of shortage. It is, therefore, difficult to generalize the over-all effect of the above modeling of obsolescence on the resulting procurement policy without examining in more detail a specific parameter value used to describe the occurrence of obsolescence for a problem in question.

CHAPTER IV: A DYNAMIC PROCUREMENT-DISPOSAL MODEL

A. Introduction

In this chapter is developed a dynamic procurement-disposal inventory model which assumes that the number of time periods until obsolescence and the demand for each line item are random variables with known probability distributions. While the model assumes that the demand distribution is known for an item for each time period of its lifetime, it allows this distribution to vary from period to period. Thus, diminishing demand toward the end of the item's lifetime may be reflected. In addition, although the probability distribution of time to obsolescence is assumed known, it too can vary from period to period. Also assumed known are: holding cost per unit per time period; stockout cost per unit per time period; unit price; fixed reorder cost; and disposal cost per unit.

Each of the known costs referred to above may be varied from period to period. Thus, any considerations which suggest that costs will change in some fashion in the future can be taken into account in computing policy.

A further restriction is imposed solely for reasons of computing simplicity. This is that lead time is equal to one inventory period. The program is capable of modification to allow lead time to be a multiple of the inventory period, but for purposes of numerical analysis it was felt that at this time the great increase in computing would not be justified

B. Some General Remarks

It should be noted that the dynamic program operates backwards in time. It begins by determining an optimal policy for some "last"[1] time period, proceeds from that to an optimal policy for the next to last period, and so on. Thus, for an obsolescence date of twenty periods into the future, the twentieth from the "last" period is the initial period and the best policy for that period is the best initial policy. A "best policy" for a period is defined by the program as follows: it is a policy which specifies for each feasible stock level left on hand at the end of the previous period what the level ought to be brought to at the beginning of this period in order that the sum of all costs from this period on shall be minimal.

The program is based on the fact that at the start of any time period the optimal stock level depends only on the inventory left from the previous period and not on how that inventory level came about; that is, the past influences the present through a single number, the inventory level at the end of the immediately preceding period. For that reason, the program is able to specify the future cost associated with starting a time period with each possible inventory level, without reference to what inventory will, in fact, be left over from the preceding period or what policy was used in the earlier periods. It only

[1] This "last" period is determined by examining the probability distribution of obsolescence occurring at time $t(t=0,\ldots,\mathcal{T},\ldots,\infty)$. We find a $\mathcal{T}$ such that the probability of obsolescence occurring after $\mathcal{T}$ is less than some arbitrary ϵ. $\mathcal{T}$ is that "last" period.

needs to assume that an optimal policy is used in all later periods. This fact, plus a knowledge of what policy is optimal during the first period after obsolescence (specifically, to dispose of all stock on hand) provides the recursive procedure of the dynamic program.

The program produces, then, a policy for each time period. Each policy is used to calculate a preceding policy, and one entry of each (the best starting stock when the inventory left from the previous period is zero) is also used to provide an optimal initial order for each possible number of periods to obsolescence.

Furthermore, since the full set of order policies include optimal stock levels when there is already some stock left over from the past, the model is fully capable of suggesting reorder decisions as well as initial order amounts.

C. A Specific Formulation of the Dynamic Program

1. Definitions

a. Cost of changing stock level

Let $\gamma_i\left[a,b\right]$ be a known function denoting the cost in period i of converting a stock level of "a" units into a stock level of "b" units. $(a,b \geq 0)$.

It is postulated that γ_i has the following properties:

(4.1) $$\gamma\left[a,b\right] + \gamma\left[b,c\right] \geq \gamma\left[a,c\right]$$

(4.2) $$\gamma\left[a+k,\ b+k\right] = \gamma\left[a,b\right] \quad \text{for all } k \text{ for which } \begin{matrix} a+k \\ b+k \end{matrix} \geq 0$$

(4.3) $$\gamma\left[a,b\right] \geq 0,\ \gamma\left[a,a\right] = 0$$

The purchase price of an item is not included in reorder cost $(\gamma[0,+])$; it is contained instead in disposal cost $(\gamma[0,-])$.

Making use of (4.2) we may define $\gamma[a,b]$ for a,b including negative values, and may then define $\theta_i(a) = \gamma[0,a] = \gamma[b, b+a]$ for all a = 0, $\pm$ 1, $\pm$ 2, ...

$\theta_i(a)$ shall be used as the input quantity.

b. Shortage cost

Let

$\text{Cost}^i_{out}(r-k)$ = Cost in <u>ith</u> period associated with an incoming order r which exceeds the beginning inventory k, r-k = 1, 2, 3 ....

This cost will be specified as an analytic function rather than a table of values in the computer program.

c. Storage cost

$\text{Cost}^i_{sto}(r,k)$ = Storage costs in <u>ith</u> period associated with an initial stock of k and an incoming order of r. If this cost is to be charged at the end of a period, it can be regarded as a function of k-r, the excess of inventory over demand. Since this cost is defined for each time period, any discounting of the future or interest charges on investment will be included in it.

d. Issue cost

Let

$\mathrm{Cost}^{i}_{iss}(r)$ = cost of issuing r units in the <u>ith</u> period.

$$r = 1, 2, 3, \ldots.$$

Note that this cost must include the unit purchase cost in order for the program to operate properly.

e. Probability Distribution of Demand

Let $\Pr(X_i = r)$ = Probability that the demand in period i is exactly r units.

$$r = 0, 1, 2, \ldots.$$

f. Probability Distribution of Obsolescence

Let a_i = Conditional probability that obsolescence occurs in period i, given that it has not occurred in periods $N, N-1, \ldots, i+1$.

More specifically

$$a_i = \frac{d_i}{\sum_{j=1}^{i} d_j}$$

where d_i is the probability that obsolescence occurs in period i, and $\sum_{j=1}^{n} d_i = 1$.

g. Expected future costs

Let

$\phi_i(K)$= Expected cost from i on, if the initial stock level in the ith period is K: $K = 0, 1, 2, \ldots$, and an optimal policy is used in period i as well as in each of the future periods.

2. Induction

Since we have an optimal policy for time period zero (i.e., if there are K units in inventory, dispose of K units) and a set of expected future costs for period zero,

$$\phi_0(K) = \theta_0(-K) \qquad \text{for} \quad K = 0, 1, 2, \ldots .$$

the entire dynamic program can be specified by describing a procedure for calculating policy and future cost in period i from policy and future cost in period $i-1$. Note that period i is the time period which is i periods earlier than the date of obsolescence.

Assume now that $\phi_{i-1}(K)$ has been calculated.

(4.4) Then:

$$\phi_i(K) = \sum_{r=0}^{\infty} Pr(x_i=r) \cdot \Psi_i(K,r)$$

And $\Psi_i(K,r)$ is given by:

(4.5) $$\Psi_i(K,r) = \text{Cost}^{\,i}_{\,out}\,(r-K) + \text{Cost}^{\,i}_{\,sto}\,(K,r) + \text{Cost}^{\,i}_{\,iss}\,(r)$$

$$+ (1-a_i)\, M\,(r-K) + a_i\, \theta_i(r-K) \qquad \text{if } r > K$$

$$= \text{Cost}^{\,i}_{\,sto}\,(K,r) + \text{Cost}^{\,i}_{\,iss}(r) + (1-a_i)M(r-K) + a_i\,\theta_i(r-K)$$

$$\text{if} \quad r \leq K$$

$M(r-K)$ is calculated as follows

For given h, $h = 0, \pm 1, \pm 2, \ldots$

(4.6) Define

$$M(h) = \min_{k \geq 0} \left[\phi_{i-1}(k) + \theta_i(h+k) \right]$$

and define $S_i(h)$ = the value of k at which $M(h)$ is achieved

$S_i(h)$ is interpreted as the ith period policy, i.e., if the initial stock for period i is K and $X_i = r$, then $S_i(r-K)$ is the amount with which to start period $(i-1)$.

Note that, in (4.5), the expression for ψ_i involves all the costs actually incurred in period i, plus the expected cost associated with the ending inventory, $M(r,K)$, if the process is to continue; and the disposal cost, $\theta(r,K)$, if the process is to terminate. The last two costs are weighted according to the probabilities of the events which will generate these costs.

Some remarks regarding the calculation of $M(h)$ are in order. Assume we start with $K \geq 0$, there is an order of $r \geq 0$ and we end the period with $h \geq 0$ in stock. Aside from costs associated with K and r above which already occur explicitly in ψ_i, we also have

CASE I $r > K$

Additional cost = $\phi_{i-1}(k) + \gamma_i\left[K-r,k\right] = \phi_{i-1}(k)$ $+ \theta_i(k+r-K)$ and to minimize this is to reorder so as to bring stock level to $S_i(r-K)$ units, and additional cost has become $M(r-K)$.

CASE II $r < K$

Additional cost = $\phi_{i-1}(k) + \gamma_i\left[K-r,k\right]$ and same minimization as in Case I.

Note that the argument of $M(h)$ is non-positive.

D. Some Remarks on Optimal Policies of Dynamic Procurement-Disposal Model

A number of computer runs were made in order to study the effect of parameter variations on the optimal procurement and disposal policies. Detailed results of these runs are presented in Appendix A.

Within the range of parameter variations explored, it was found that when a set-up cost is involved in ordering and disposing of the stock, the optimal ordering and disposal policies are of an (s,S) type. That is to say there are four uniquely determined integers $K_0 < K_1 < K_2 < K_3$ such that if the initial inventory level, say H, is less than K_0, an order is placed to bring the inventory level up to K_1; if the inventory condition is $K_0 \leq H \leq K_3$, no action is taken; if the inventory level exceeds K_3, it is disposed down to K_2. It was observed that these policy parameters (K's) are related to the expected total inventory cost function $\phi(K)$ (See pp.) in the following manner:

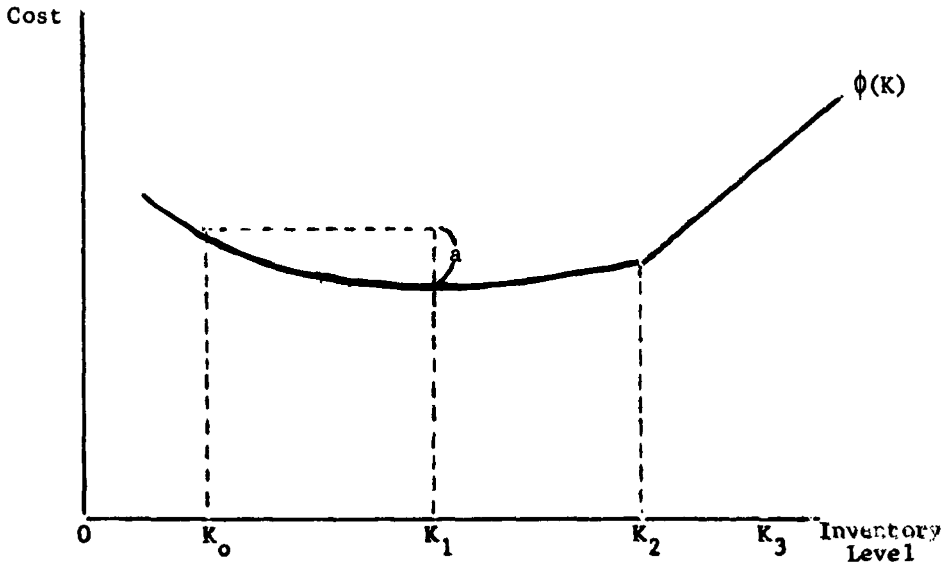

Fig. 1: A Typical Expected Total Inventory Cost Function

a = fixed ordering cost

b = salvage value obtained from disposing one unit of the stock

c = unit storage cost

d = fixed disposal cost

1) K_1 is an integer value of K which minimizes $\phi(K)$

2) K_0 is the largest integer such that

$$\phi(K_0) > \phi(K_1) + a \quad \text{and} \quad K_0 < K_1$$

3) K_2 is an integer such that

$$\phi_-'(K_1) < b < \phi_+'(K_1) \quad \text{and} \quad K_2 > K_1$$

$\phi_-'(K_1)$ and $\phi_+'(K_1)$ denote the left-hand and right-hand derivatives at K_1 respectively.

4) K_3 is an integer such that

$$C(K_3-K_2) > d \quad \text{and} \quad K_3 > K_2$$

These findings seem to make sense for the following reasons:

1) Since the objective of the model is to minimize the total expected inventory cost, it is natural to bring the inventory level up to K_1.

2) If a set-up cost is involved in ordering, the cost function in effect between K_0 and K_1 is the dotted line above the $\phi(K)$ function in Fig. 1. It is then most economical to order from K_0.

3) Suppose there is no fixed disposal cost. As the initial inventory level increases, the total expected inventory cost will also rise because of an increase in shortage cost. At some point, namely K_2, it becomes cheaper to dispose an additional unit of the stock than to retain it since in this model a salvage value is attached to a unit disposed.

4) If a set-up cost is incurred for disposing one unit of the stock, then the cumulative storage costs must exceed the fixed disposal costs before a disposal becomes economically attractive.

Although an (s,S) type policy seems to characterize all the optimal policies obtained in this study (see Appendix A), even when a constant fixed cost is involved in ordering and disposing of the stock, it should be noted that such a policy cannot be optimal in all cases. However, it is probably safe to conclude that in most practical situations an (s,S) policy will produce a near optimal solution.

If, in fact, the item then survives until t_1, the probability that it will survive until t_2, $t_1 < t_2 < T$ would be

$$F_2(t_2)=F_1(t_2|t_1)=\frac{\frac{1}{T}\int_{t_1}^{t_2} dx}{1-\frac{1}{T}\left[\int_0^{t_0} dx + \int_0^{t_2} dx\right]} = \frac{t_2 - t_1}{T-t_0-t_2}$$

$$F_n(t_n)=F_{n-1}(t_n|t_{n-1}) = \frac{\frac{1}{T}\int_{t_{n-1}}^{t_n} dx}{1-\frac{1}{T}\left[\int_0^{t_0} dx + \int_0^{t_{n-1}} dx\right]} = \frac{t_n-t_{n-1}}{T-t_0-t_{n-1}}$$

Clearly, this recursion is time-dependent. Similarly, if we assume that $\log\ t$ is Cauchy-distributed, we find that

$$F_n(t_n|t_{n-1}) = \frac{1}{\pi}\left[\frac{\tan^{-1}(\log t_n-\mu) - \tan^{-1}(\log t_{n-1}-\mu)}{1+2\pi(1+\frac{m}{2}) - \tan^{-1}(\log t_n-\mu)-\tan^{-1}(\log t_{n-1}-\mu)}\right]$$

equally, a time-dependent recursion.

We now consider some examples of distributions which are reproducible under the operation of taking the conditional. The simplest example of this property is, of course, furnished by the exponential distribution, with probability of survival to age t equal to e^{-kt}; after survival to age t_0 to the probability of surviving for a further interval of t is again e^{-kt}. We present three related families of mortality distributions (including the exponential as a special case) which exhibit reproducible conditional distributions.

If the cumulative distribution function $F(t)$ is represented in terms of the age-specific death rate $\lambda(t)$, we have

$$F(t) = 1 - \exp\left[-\int_0^t \lambda(u)\,du\right]$$

The conditional probability $F(\tau|t_0)$ of surviving an additional time τ, after surviving to t_0, is given by

$$\frac{F(t_0+\tau) - F(t_0)}{1-F(t_0)} = 1 - \exp\left[-\int_{t_0}^{t_0+\tau} \lambda(u)\,du\right]$$

The particular families described below result from assuming that $F(\tau|t_0)$ is related to $F(t)$ by a transformation of the form $t \rightarrow k\tau$ and $\lambda \rightarrow h\lambda$. If λ is differentiable no other families can be reproducible under the same transformations (demonstration omitted)

Case I: $\lambda(t) = \alpha e^{\beta t}$; $k = 1$, $h = e^{\beta t_0}$

$$1 - \exp\left[-\int_{t_0}^{t_0+\tau} \lambda(u)\,du\right] = 1 - \exp\left[-\alpha\int_{t_0}^{t_0+\tau} e^{\beta u}du\right]$$

$$= 1 - \exp\left[-\frac{\alpha}{\beta}\left\{e^{\beta(t_0+\tau)} - e^{\beta t_0}\right\}\right]$$

$$= 1 - \exp\left[-\frac{\alpha e^{\beta t_0}}{\beta}\left\{e^{\beta\tau}-1\right\}\right]$$

(5.1) $$= 1 - \exp\left[-\alpha e^{\beta t_0}\int_0^{\tau} e^{\beta u}\, du\right]$$

Case II: $\lambda(t) = \alpha(1+\beta t)^{\gamma}$; $k = \frac{1}{1+\beta t_0}$, $h = (1+\beta t_0)^{\gamma+1}$

$$1 - \exp\left[-\int_{t_0}^{t_0+\tau} \lambda(u)\, du\right] = 1 - \exp\left[-\alpha\int_{t_0}^{t_0+\tau} (1+\beta u)^{\gamma}\, du\right]$$

<u>IIa: $\gamma \neq -1$</u>

$$1 - \exp\left[-\alpha\int_{t_0}^{t_0+\tau} (1+\beta u)^{\gamma}\, du\right]$$

$$= 1 - \exp\left[\frac{-\alpha}{\beta(\gamma+1)}\left\{(1+\beta\,(t_0+\tau))^{\gamma+1} - (1+\beta t_0)^{\gamma+1}\right\}\right]$$

$$= 1 - \exp\left[-\frac{\alpha}{\beta(\gamma+1)}(1+\beta t_0)^{\gamma+1}\left\{(1+\frac{\beta\tau}{1+\beta t_0})^{\gamma+1} - 1\right\}\right]$$

(5.2) $$= 1-\exp\left[-\alpha(1+\beta t_0)^{\gamma+1}\int_0^{\frac{\tau}{1+\beta t_0}} (1+\beta u)^{\gamma}\, du\right]$$

IIb: $\gamma = -1$

$$1 - \exp\left[-\alpha \int_{t_0}^{t_0+\tau} (1+\beta u)^{-1}\, du\right]$$

$$= 1 - \exp\left[-\frac{\alpha}{\beta} \ln \frac{1+\beta(t_0+\tau)}{1+\beta t_0}\right]$$

$$= 1 - \exp\left[-\frac{\alpha}{\beta} \ln \left(1 + \frac{\beta\tau}{1+\beta t_0}\right)\right]$$

$$(5.3) = 1-\exp\left[-\alpha \int_{0}^{\frac{\tau}{1+\beta t_0}} (1+\beta u)^{-1}\, du\right]$$

Cumulative distribution functions

Integrating out the age-specific death rate in (5.1), (5.2), and (5.3), and reparameterizing slightly yields the following cumulative distributions:

Case I: $F(t) = 1-e^{-a(e^{bt}-1)}$

The conditional reproducibility of this function may be demonstrated as follows:

Let $F(t_1|t_0)$ denote the probability of surviving to t_1 given that it has survived up to t_0.

$$F(t_1|t_0) = \frac{1-e^{-a(e^{bt_1}-1)} - 1 + e^{-a(e^{bt_0}-1)}}{e^{-a(e^{bt_0}-1)}}$$

$$= 1-e^{-ae^{bt_0}(e^{b(t_1-t_0)}-1)}$$

By defining a new variable

$$\tau = t_1 - t_0$$

and new parameters

$$a' = ae^{bt_0} \quad \text{and} \quad b' = b,$$

We obtain a new distribution

$$G(\tau) = 1-e^{-a'(b'\tau-1)}$$

which has the identical form as the original one.

Similarly, the reproducibility of the distributions in Cases IIa and IIb can be demonstrated.

Case IIa: $F(t) = 1-e^{-a\left[(1+bt)^c-1\right]}$; $a' = a(1+bt_0)^c$,

$$b' = \frac{b}{1+bt_0}$$

$$c' = c$$

Case IIb: $F(t) = 1-(1+bt)^{-c}$; $b' = \frac{b}{1+bt_0}$, $c' = c$

Note that Cases I and IIb are both limiting cases of IIa. The exponential distribution is also a limiting case of IIa.

$$F(t_1|t_0) = \frac{1-e^{-a(e^{bt_1}-1)} - 1 + e^{-a(e^{bt_0}-1)}}{e^{-a(e^{bt_0}-1)}}$$

$$= 1-e^{-ae^{bt_0}(e^{b(t_1-t_0)}-1)}$$

By defining a new variable

$$\tau = t_1 - t_0$$

and new parameters

$$a' = ae^{bt_0} \text{ and } b' = b,$$

We obtain a new distribution

$$G(\tau) = 1-e^{-a'(b'\tau-1)}$$

which has the identical form as the original one.

Similarly, the reproducibility of the distributions in Cases IIa and IIb can be demonstrated.

Case IIa: $F(t) = 1-e^{-a\left[(1+bt)^c-1\right]}$; $a' = a(1+bt_0)^c$,

$$b' = \frac{b}{1+bt_0}$$

$$c' = c$$

Case IIb: $F(t) = 1-(1+bt)^{-c}$; $b' = \frac{b}{1+bt_0}$, $c' = c$

Note that Cases I and IIb are both limiting cases of IIa. The exponential distribution is also a limiting case of IIa.

The properties exhibited by this very general class of functions are made use of in the simpler of the two Bayesian-dynamic models developed subsequently. In the example presented, the simplest a priori assumption is made, namely that the probability of survival to time t is exponential. However, most models could just as well involve more general functions which are members of this class.

In subsequent chapters we develop these models.

CHAPTER VI: INVENTORY MODELS WITH MARKOV DEMAND

A. Introduction

In this chapter, we present some inventory models which are useful for decision-making regarding ordering and disposal activities involving a single item. Policies considered are of the well-known (s,S) type. For ordering, whenever the stock level falls below a certain level s, enough new stock is ordered to bring the level up to another prespecified level S; if the stock level exceeds s, no order is placed. As to disposal, the policy operates as follows: If the stock level exceeds a certain level D, a disposal takes place in order to bring the stock level down to a new level d; when the level is short of D, no disposal is made.

In Section 2, a demand process is formulated. We assume that the system which generates demand can be in any of several states in a given time interval. Each state has its own demand pattern. Hence, the demand in each period, which is a random variable, may or may not be identically distributed in successive periods. At the end of each time interval, we have data on the demand for that interval; based on these data we then efficiently forecast the demand for the future by means of Bayes' formula.

In Section 3, a general inventory model which is embedded in the demand process described in the previous section is presented with a computation procedure. A specific example of such a model is given with some calculated results. Finally, a very special case of the general model is discussed. This last model is distinguished by the property that when the system has made a transition to its terminal state, which is characterized by zero demand, it stays there. In Section 4, two schemes which can be used for improving the accuracy of predicting the demand for the future are introduced.

B. The Demand Process

1. Markov Demand Generation

Of course, the system which underlies the demand generation of even a single item in the vast Navy supply system is quite complex. Fortunately, a Markov process provides a manageable and quite general mathematical model for our analytical study.

The basic concepts of the Markov processes are those of "states" of a system and state "transition." We say a system is in a certain state when it is completely describable by the values of parameters which define that state. A system is said to have made a transition from one state to another when the parameters which describe it have changed from the values specified for one state to those for another.

Consider a demand generating system which can be in one of a finite number of states, S_r where $r = 1,2,\ldots,n$, at any time. Each state, in general, is assumed to have a different demand pattern which is characterized by a parameter such as mean demand rate. This system makes state transitions according to a certain transition probability matrix $||P_{rs}||$. Its typical element P_{rs} stands for the probability of the system making a transition from S_r to S_s. Furthermore, we use a vector (π_r) to denote the *a priori* state probability distribution. Its typical element π_r is the probability that the system is initially in S_r and $\sum_r \pi_r = 1$. Let $g_r(x)$ denote the conditional distribution of demand X given that the system is in S_r.

The above finite state discrete time process is the scheme incorporated in the subsequent inventory models to represent the demand generation for a single item. This process can also readily handle the situation where demands for several items are linked through underlying states. All one must do is specify a conditional joint distribution of item demands for each state. One of the models to be discussed later makes use of this notion of linked demands.

2. Prediction of Demand by Bayes' Formula

Suppose we have observations on demand X in this period and we wish to estimate the demand for the next period. How shall we proceed? One method of prediction is to calculate *a posteriori* state probabilities by means of Bayes formula. Then combining these state probabilities with the transition probabilities provides a basis for estimation.

We are given the *a priori* state probabilities $\pi_r = \Pr\left[S = S_r\right]$ for $r = 1,2,\ldots,n$ and we wish to calculate the *a posteriori* state probabilities $\pi'_r = \Pr\left[S = S_r \mid X = x\right]$, for $r = 1,2,\ldots,n$, after observing $X = x$. Using the definition of conditional probability

$$\pi'_r = \Pr\left[S = S_r \mid X = x\right],$$

$$(6.1)\qquad = \frac{\Pr\left[S = S_r \text{ and } X = x\right]}{\Pr\left[X = x\right]}$$

The numerator of (6.1) can be evaluated as follows:

$$(6.2)\quad \Pr\left[S = S_r \text{ and } X = x\right] = \Pr\left[X = x \mid S = S_r\right]\cdot\Pr\left[S = S_r\right],$$

$$= g_r(x)\cdot\pi_r.$$

To evaluate the denominator we note that the possible outcomes leading to $X = x$ are (S_1,x), (S_2,x), . . . (S_n,x) Hence

$$(6.3)\qquad \Pr\left[X = x\right] = \sum_{j=1}^{n} \Pr\left[X = x \text{ and } S_j\right],$$

$$= \sum_{j=1}^{n} g_j(x)\cdot\pi_j.$$

From (6.2) and (6.3), we can then calculate (6.1).

$$(6.4)\qquad \pi'_r = \frac{g_r(x)\cdot\pi_r}{\sum_{j=1}^{n} g_j(x)\cdot\pi_j}.$$

Making use of knowledge about the a posteriori state probability and the state transition probability matrix $||P_{rs}||$, we then obtain the a priori state probability (π''_r) for the next period.

(6.5) $$\pi''_s = \sum_{r=1}^{n} P_{rs}\, \pi'_r,$$

$$= \frac{\sum_{r=1}^{n} P_{rs} \cdot g_r(x) \cdot \pi_r}{\sum_{j=1}^{n} g_j(x) \cdot \pi_j} .$$

(π''_s) gives us a basis for predicting the future states of the system on the basis of all observable demands up to the present, by recursive application of (6.5) after each time period. Furthermore, it also makes use of available a priori information regarding the future states of the system. For instance, so called program data such as the expected numbers of flight hours and the expected obsolescence rate may be translated into the form of the a priori state probabilities (π_r) and the state transition probabilities $||P_{rs}||$. In this way, the empirical evidence can be supplemented by the judgment of the operating people to obtain the most efficient prediction results.

C. General Markov Inventory Model

1. Formulation of the Model

The problem considered here is that of stocking a supply of one item to meet an uncertain demand which is assumed to be generated by a Markov process described in the previous section. Various costs associated with oversupply and undersupply are assumed to be operative. Orders are placed at the beginning of each time period of equal length. The orders are assumed to be fulfilled either immediately or are delivered one time interval later. After the order has been placed, a demand is made. This demand is satisfied from the existing inventory, with excess demand leading to a penalty cost. Unfilled demand can be assumed to be either lost or backlogged[1]

Let us first introduce the following notation:

ψ = policy

α = action according to policy ψ, presumably $\alpha = \psi(Y,(\pi))$

A = the set of admissible actions, $\alpha \in A$

Y = inventory vector, describing the inventory status at the beginning of a period

X = demand vector

π_r = a priori state probability

$L(Y,x,\alpha)$ = expected current cost function with initial inventory Y, demand x and taking action α.

$C_\psi(Y,(\pi_r))$ = expected total cost of choosing α according to ψ and process beginning with Y and (π_r).

The optimization problem involves finding ψ to minimize $C_\psi(Y,(\pi_r))$ for all $(Y,(\pi_r))$. We note the following recursive relation:

$$(6.6)\quad C_\psi(Y,(\pi_r)) = \sum_{s,x} \pi_s g_s(x) \left[L(Y,x,\alpha) + \rho C_\psi (Y',(\pi_s''))\right] ,$$

[1]As long as this assumption is maintained, the computation of optimal policies for any constant lead time problem is still manageable. As to the inventory problem with a stochastic lead time, the literature is rather scarce on this subject except under some simplified assumptions; clearly, further exploration is needed in this area of inventory study.

where

Y' = a vector describing terminating inventory status for this period (or initial inventory status for next period). It assumes the following functional relation: $Y' = h\ (Y, x, \alpha)$

π''_s = *a priori* state probability for next period. This was derived in the previous section, as a function of (π_s) and x.

ρ = a discount factor, $0 < \rho < 1$. The introduction of such a discount factor prevents infinite costs from entering.

Following the well known optimality principle, the optimum total cost function may be stated as follows:

$$(6.7)\quad C_\psi(Y,(\pi_r)) = \underset{\alpha}{\mathrm{Min}} \sum_{s,x} \pi_s g_s(x) \left[L(Y,x,\alpha) + \rho\, C_\psi(Y',(\pi''_s))\right]$$

If ψ is an optimum policy.

This functional equation can be solved by the following iterative method: Select an arbitrary set of starting policies and cost functions, say ψ_o and $K_o\ (Y,(\pi_r))$. Then recursively calculate sequences of policies $\langle\psi_n\rangle$ and cost functions $\langle K_n(Y,(\pi_r))\rangle$ such that

$$(6.8)\quad K_{n+1}(Y,(\pi_r)) = \underset{\alpha}{\mathrm{Min}} \sum_{s,x} \pi_s g_s(x) \left[L(Y,x,\alpha) + \rho\, K_n(h(Y,x,\alpha),(\pi''_s))\right]$$

and $\psi_{n+1}\ (Y,(\pi_r))$ is the minimizing α for each $(Y,(\pi_r))$. When these sequences converge, the limits of the sequences are the solution to (6.7). In our numerical study of this model, it was found that the convergence of the cost function sequence $\langle K_n \rangle$ could be painfully slow, expecially when the values of starting cost functions are poor approximations of the true cost functions or the discount factor ρ is close to unity. However, it was found that the policy sequence $\langle \psi_n \rangle$ converges much more rapidly.

2. Computational Short-cut

We note that in any minimization or maximization problem of this type what matters is the pattern of differences among the relevant variables, rather than absolute levels. Making use of this idea, we are then able to reduce the number of iterations necessary for the sequence $\langle K_n \rangle$ to converge.

Let $(\overset{o}{Y},(\overset{o}{\pi}_r))$ be any particular value of $(Y,(\pi_r))$, and define

$$(6.9) \qquad \bar{C}_\psi \; (Y,(\pi_r)) = C_\psi \; (Y,(\pi_r)) - C_\psi \; (\overset{o}{Y},(\overset{o}{\pi}_r)),$$

where ψ is optimum policy and C_ψ corresponding minimum cost.

Substituting (6.9) into (6.7), we have

$$(6.10) \qquad \bar{C}_\psi(Y,(\pi_r)) = \underset{\alpha}{\mathrm{Min}} \sum_{s,x} \pi_s g_s(x) \left[L(Y,x,\alpha) + \rho \, \bar{C}_\psi(h(Y,x,\alpha),(\pi_s'')) \right]$$

$$- (1-\rho) \; C_\psi \; (\overset{o}{Y},(\overset{o}{\pi}_r)).$$

This subtraction of a constant will in no way affect the minimization. Hence, we set $C_\psi(\overset{o}{Y},(\overset{o}{\pi}_r)) = 0$. We then have another expected total cost function, involving only the cost differences.

$$(6.11) \quad \bar{C}_\psi\, (Y,(\pi_r)) = \underset{\alpha}{\mathrm{Min}}\ \sum_{s,x} \pi_s g_s(x) \left[L(Y,x,\alpha) + \rho\, \bar{C}_\psi(h(Y,x,\alpha),(\pi''_s))\right]$$

The new iterative procedure is exactly the same as before except after each iteration the following subtraction is made to set $K_{n+1}(\overset{o}{Y},(\overset{o}{\pi}_r)) = 0$

$$(6.12) \quad K_{n+1}(Y,(\pi_r)) = \underset{\alpha}{\mathrm{Min}}\ \sum_{s,x} \pi_s g_s(s) \left[L(Y,x,\alpha) + \rho\, K_n(h(Y,x,\alpha),(\pi''_r))\right]$$

$$-\underset{\alpha}{\mathrm{Min}} \sum_{s,x} \overset{o}{\pi}_s g_s(x) \left[L(\overset{o}{Y},x,\alpha) + \rho\, K_n(h(\overset{o}{Y},x,\alpha),(\pi''_r))\right]$$

When convergence of the sequence $\{K_n\}$ is satisfactory, the last subtractive constant is divided by $(1-\rho)$ to obtain $C_\psi(\overset{o}{Y},(\overset{o}{\pi}_r))$.

$$(6.13) \quad C_\psi\, (\overset{o}{Y},(\overset{o}{\pi}_r)) = \frac{\underset{\alpha}{\mathrm{Min}} \sum_{s,x} \overset{o}{\pi}_s g_s(x) \left[L(\overset{o}{Y},x,\alpha) + \rho\, K_n(h(\overset{o}{Y},x,\alpha),(\pi''_r))\right]}{1-\rho}$$

This is a quick way of summing the infinite series $(1+\rho+\rho^2+\ldots)$ Finally, $C_\psi(Y,(\pi_r))$ for other $(Y,(\pi_r))$ are calculated by means of (6.9)

By adopting this short-cut method, a substantial reduction in computation time was observed.

3. Numerical Examples

In this numerical example, a demand-generating system consisting of two states, one having a high demand rate, and the other a low demand rate, is considered. Orders are made at the beginning of each regularly spaced period and a delivery lag of one period is considered. Any unfilled demand for current period becomes demand for next period. The conditional distribution of demand is assumed to be Poisson.

Definition of symbols used and parameter values assumed are given below:

y = initial inventory level. Both positive and negative values are allowed. If y assumes a negative value, it means that there are unfilled demands.

α = desired inventory level. The difference $\alpha - y$ represents the optimal order quantity, if it is positive. If the difference is negative, it represents the optimal disposal quantity. In the first example, we are considering $\alpha \geq \max(y,0)$, i.e., no disposal activities are allowed. Of course, it presents no problem if one wishes to allow disposal activities as in the second example; all that needs to be done is to set the domain of α to be $\alpha \geq 0$. If we allow α to assume negative values it means that the stock can be returned to the suppliers and credit received for it.

x = demand per period

π = *a priori* probability that the system is in state 1. Then $(1-\pi)$ is *a priori* probability that the system is in state 2.

$g_1(x)$ = probability distribution of demand when the system is in state 1

$$e^{-2} \frac{2^x}{x!}$$

$g_2(x)$ = probability distribution of demand when the system is in state 2

$$e^{-0.4} \frac{(0.4)^x}{x!}$$

$h_\pi(x)$ = probability distribution of x weighted by a *priori* state probabilities

$$h_\pi(x) = \pi\ g_1(x) + (1-\pi)\ g_2(x)$$

d_1 = unit storage cost, 0.5

d_2 = unit out-of-stock cost, 5.0

d_3 = fixed order cost, 1.0

d_4 = unit order cost, 0.5

$||P_{rs}||$ = transition probability matrix, $r,s, = 1,2$

r \ s	1	2
1	0.7	0.3
2	0.1	0.9

π'' = a *priori* probability that the system will be in state 1 in the next period.

$$\pi'' = \frac{p_{11}\ \pi\ g_1(x) + p_{21}\ (1-\pi)\ g_2(x)}{\pi\ g_1(x) + (1-\pi)\ g_2(x)}$$

ρ = discount factor, 0.99

With the above notation, we now can proceed to specify the expected total cost function which is composed of the expected current costs and discounted future cost.

Expected holding and shortage costs, $L_1(y,\pi)$, to be charged during the current period, assuming that an order will not be delivered until next period is:

$$(6.14)\quad L_1(y,\pi) = \begin{cases} d_1 \sum_{x=0}^{y} (y-x)\, h_\pi(x) + d_2 \sum_{x=y+1}^{\infty} (x-y)\, h_\pi(x) & y \geq 0 \\ d_2\,(-y) + d_2 \sum_{x=0}^{\infty} x\, h_\pi(x) & y < 0 \end{cases}$$

Suppose the ordering cost, $L_2(y,\, \alpha)$, is charged when orders are placed, it can be expressed as follows:

$$L_2(y,\, \alpha) = \begin{cases} d_3 + d_4\,(\alpha - y) & \alpha > y \\ 0 & \alpha = y \end{cases}$$

Then the expected current cost function is:

$$L(y,\, \pi,\, \alpha) = L_1(y,\, \pi) + L_2(y,\, \alpha)$$

Let $C_\psi\,(y,\pi)$ represent the total expected cost if the provisioning is done optimally.

$$(6.15)\quad C_\psi\,(y,\pi) = \min_{\alpha > \max(y,0)} \left[L(y,\pi,\alpha) + \rho \sum_{x=0}^{\infty} h_\pi(x)\, C_\psi\,(\alpha - y, \pi'') \right]$$

The solution to (6.15) was obtained by means of the iterative procedure described in the previous section and is presented in Table 1. Note that the condition $\alpha > \max(y,0)$ restricts the policy from carrying out disposal activities.

Another set of calculations was performed by introducing disposal activities in the above problem and by reducing the unit stockout cost. (Rest of the parameters were unchanged.) More specifically,

TABLE 1: OPTIMUM POLICY TABLE WITHOUT DISPOSAL*

a priori state probability

Initial Inventory Level y \ π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
7	7	7	7	7	7	7	7	7	7
6	6	6	6	6	6	6	6	6	6
5	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	6	6	6
3	3	5	5	5	5	5	6	6	6
2	4	5	5	5	5	5	6	6	6
1	4	5	5	5	5	5	6	6	6
0	4	5	5	5	5	5	6	6	6
-1	4	5	5	5	5	5	6	6	6
-2	4	5	5	5	5	5	6	6	6

*For a given a priori state probability and initial inventory level, we read off an appropriate entry in the above table. The difference between this entry and the initial inventory level is the optimum order quantity. When the difference is zero, no action should be taken. For all $y < -2$, the optimum policy repeats itself.

we set the domain of minimization to $\alpha > 0$ in (6.15) and set $\alpha_2 = 25$. Results are tabulated in Table 2.

TABLE 2: OPTIMUM POLICY TABLE WITH DISPOSAL

a priori state probability

y \ π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
7	3	7	7	7	7	7	7	7	7
6	3	6	6	6	6	6	6	6	6
5	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	5	5	6
2	2	4	4	4	4	4	5	5	6
1	3	4	4	4	4	4	5	5	6
0	3	4	4	4	4	4	5	5	6
-1	3	4	4	4	4	4	5	5	6
-2	3	4	4	4	4	4	5	5	6

4. A Special Case of the General Model

The example discussed in this section is a special case of the general model. It is of special interest to our study of obsolescence because:

1. One of the states is assumed to have zero demand. Once the system is in this state, it is not possible to make transitions to other states. Hence, we consider that obsolescence

has occurred when the system has entered the zero demand state.

2. It illustrates how those failure or mortality distributions with the property of "conditional reproducibility" which have been discussed in an earlier chapter, enter into the general Markov inventory model. With the introduction of such a distribution into the model, it is appropriate to look for optimum policies which depend on the inventory level and on some function of the past observable demands but not explicitly dependent on time.

Consider a system with two underlying states. When the system is in State 1, the item demand follows the conditional distribution $g_1(x)$ for $x=0, 1,2,\ldots$ When it is in State 2, the conditional demand function $g_2(x)$ is defined to be

$$g_2(x) = \begin{cases} 1 & \text{for } x = 0 \\ 0 & \text{for } > 0 \end{cases}$$

Then, in the language of the previous section, the probability distribution of X weighted by a priori state probabilities is

$$h_\pi(x) = \begin{cases} \pi\, g_1(x) + (1-\pi) & \text{for } x = 0 \\ \pi\, g_1(x) & x > 0 \end{cases}$$

Suppose transitions are made according to the following transition probabilities

r \ s	1	2
1	e^{-k}	$1-e^{-k}$
2	0	1

The *a priori* probability that the system will be in State 1 in the next period given that the current demand is $X = x$

$$\pi'' = \begin{cases} \dfrac{e^{-k}\,\pi\, g_1(x)}{\pi\, g_1(x) + (1-\pi)} & \text{for } x = 0 \\ e^{-k} & x > 0 \end{cases}$$

With these definitions of $h_\pi(x)$ and π'', we can formulate the total expected cost function similar to (6.15) and solve it.

Note that e^{-k} is the probability that the system survives in any given period given that it did not terminate during the immediately preceding period if we assume that the system's survival follows an exponential distribution. This can be shown as follows:

Let t = a random variable which represents the age of a system, $t \geq 0$

T = the age of the system when it terminates, $T \geq 0$

$f(t)dt = Pr(t < T < t + dt)$, The probability that the system will terminate in the interval t to $t + dt$

$Pr(T \leq t)$ = the probability that the system will terminate in the interval 0 to t.

$Pr(t \leq T \leq t+dt | T > t)$ = The probability that the system will terminate in an additional time interval dt given that it did not terminate before t.

Suppose we assume $f(t) = ke^{-kt}$. Then

$$Pr(T \leq t) = 1 - e^{-kt}$$

$$Pr(t \leq T \leq t + dt | T > t) = 1 - e^{-k(dt)}$$

Now the probability that the system will survive the additional time interval dt given that it has survived up to t is

$$1 - Pr(t \leq T \leq t + dt | T > t) = e^{-k(dt)}$$

If we set dt to be unit time, we have the desired probability e^{-k}.

D. Some Extensions of the General Model

1. A Model with Linked Demands

The general model is extended to handle linked demands, in which linkage is based on underlying state S_r for $r = 1,2,\ldots,n$, in the following manner. With each state we associate a joint density function of demands such that when the system is in, say, S_s the resulting dmeands are considered to be observations based on $g_s(x_1, x_2, \ldots, x_\ell)$.

If the information such as that mentioned above can be incorporated into our model, it will contribute to the statistical determination of where the system is in any one period. For instance, if we know _a priori_ that demands for items A and B occur together in state i but in state j there is demand only for item A, and then if there are observable demands for item B, we will conclude that the system is most likely to be in state i.

With the introduction of linked demands, the calculations of the _a posteriori_ state probability (π_r') and the new _a priori_ state probability (π_r'') (as in (6.4) and (6.5) will be modified as follows:

$$\pi_r' = \frac{g_r(x_1, x_2, \ldots, x_\ell)\pi_r}{\sum_{j=1}^{n} g_j(x_1, x_2 \ldots, x_\ell)\pi_j},$$

$$\pi_s'' = \frac{\sum_{r=1}^{n} P_{rs}\, g_r(x_1, \ldots x_\ell)\pi_r}{\sum_{j=1}^{n} g_j\, (x_1, \ldots x_\ell)\pi_j}.$$

As to optimization, it will be carried out for one item at a time. Necessary modifications are quite straightforward.

Suppose we are interested in optimization of the _ith_ item. The expected total cost function can be stated as follows: (Note all the variables have the same meaning as before except now they refer to the _ith_ item).

(6.16) $$C_\psi(y_i, \{\pi_r\}) = \underset{\alpha_i}{\mathrm{Min}} \sum_{s,x} \pi_s m_s(x_i)\left[L(y_i, x_i, \alpha_i) + \rho C_\psi(y_i', \{\pi_s''\})\right]$$

where $m_s(x_i)$ is the marginal distribution of x_i

$$m_s(x_i) = \int_0^\infty \cdots\cdots \int_0^\infty g_s(x_1 \ldots x_\ell)\, d\, x_1 \ldots dx_{i-1}\, dx_{i+1} \ldots dx_\ell .$$

Again, this functional equation can be solved by the same iterative procedure described earlier.

2. A Model with Non-demand Ancillary Information

The general model can also include non-demand ancillary information which is indicative of state of the system. This will be illustrated with the following example.

Consider a three-state system in which the first two states have the same demand pattern but different state transition probabilities. The third state follows its own demand pattern and transition probabilities different from the first two states.

In such a case, since observable demands alone cannot distinguish the first state from the second state, observations on non-demand variables which have probability distributions dependent on the state will be useful for sharper statistical discrimination.

Consider the following example: the ancillary information available is about conditional distribution of a non-demand variable T.

$$\Pr\left\{T = t \mid S = S_r\right\}, \qquad \text{where } t=1,2,3 \text{ and } r=1,2,3.$$

This distribution may be assumed to look like the following table:

Prob. of T

Nature of State	1	2	3
S_1	0.95	0	0.05
S_2	0.1	0.7	0.2
S_3	0	0	1

Each row defines a distribution of T in the corresponding state. The variable T can be interpreted as being a more or less accurate indicator of the demand state. Such information is believed to be readily available within the Navy Supply System at this time.

Next, given the conditional probability $\Pr[T = t | S = S_r]$, it is of interest to know what is the a *posteriori* probability $\Pr[S = S_r | T = t]$ which is the conditional distribution of the states of the system given the ancillary information $T = t$. This can be calculated readily from the definition of conditional probability.

$$\Pr[S = S_r | T = t] = \frac{\Pr[S = S_r \text{ and } T = t]}{\Pr[T = t]}$$

$$= \frac{\Pr[T = t | S = S_r] \cdot \Pr[S = S_r]}{\sum_i \Pr[T = t \text{ and } S = S_i]}$$

$$= \frac{\Pr[T = t | S = S_r] \cdot \Pr[S = S_r]}{\sum_i \Pr[T = t | S = S_i] \cdot \Pr[S = S_i]}$$

Let $\Pr[S = S_r | T = t] = \pi_r\ (T)$

$\pi_r(T)$ will then replace the *a priori* state probabilities π_r (p.58) in the calculations of the *a posteriori* state probabilities $Pr\{S = S_r | X = x\}$, and the *a priori* state probabilities of next period $\sum_r p_{rs} Pr\{S = S_r | X = x\}$.

Optimization follows exactly the same steps as in the general model.

CHAPTER VII

SOME RESULTS OF A STATISTICAL STUDY OF OBSOLESCENCE DATA

A. Introduction

In order to generate optimal inventory decision rules from some of the models described earlier in this report (see Chapters IV and VI), it is necessary to assume some *a priori* probability distribution of the occurrence of obsolescence of an item. While it may be true that incorrect assumptions regarding the probability distribution of obsolescence is self-correcting in the long run, as we accumulate more experience with the item, the fact that we are dealing with a process which is only finite in duration suggests that the more we know *a priori* the more likely are the decisions to be optimal. We are also interested in enhancing our understanding of the factors which are associated with the process of obsolescence, i.e., we wish to determine causal factors in the process which set an obsolescence date; if this proves impossible, we wish to be able to at least isolate those factors which are related to the process. Therefore, data bearing on life-spans of a sample of items (mainly airframe and engine parts), along with a variety of economic and technical data about each of the items was examimed.

The items were first sorted according to Federal Stock Code (FSC) and Technical Supply Maintenance Code (TSMC). Within these groups we studied: (1) the frequency distribution of time-to-obsolescence, and (2) the degree of associations between some economic and technical variables and life-spans.

We learned that a certain type of the failure distributions appears to describe some of the observed data adequately and that there is a statistically significant relation between the life spans of the items in some groups and certain factors associated with the items. However, the data examined indicated that there was no single frequency distribution that may be usefully employed across the board to describe the pattern of the occurrence of obsolescence. This emphasizes the difficulty of predicting when obsolescence will take place.

B. Data Analyzed

Before entering into the discussion of the results of the statistical analysis, we shall describe in this section the data used in this study and their limitations.

The data analyzed were obtained primarily from the Aviation Supply Office (ASO). Initially a list of about 8,000 items which were declared obsolete during the first half of 1961 was prepared by ASO for this study. Unfortunately, this list was not very useful because most of the information relating to these items was no longer available when the data collection started in September 1961. An alternative list of more recent obsolescence items was then compiled from the Stock Number Data Section (SNDS) catalogs issued in August and September of 1961. These items were declared obsolete during the 13-week period ending with the date of SNDS.

The additional sources of information pertaining to the behavior of the obsolescence items in SNDS were found in the following documents: The Retention and Disposal Listing, the Parts List Catalog, the Purchase Order History Cards (also called 7-30 cards or On-Order Cards), the Federal Supply Classification Numeric Index of Classes, the Naval Aircraft Maintenance Program Glossary, and some specially prepared lists matching Navy contract numbers to their dates of issues.

The data obtained from the above documents are by no means complete. For instance: the Retention and Disposal Listing includes only data on items with surplus stock prior to the declaration of obsolescence; the parts list catalogs do not list all the applications for some of the items; between 25% and 75% of the On-Order cards for obsolete items were missing from the files. The variation in missing cards seems, in large part, to reflect the efficiency and/or prudence of the clerks responsible for them. There is a policy of purging cards from the files of those items which have not had purchase orders in the preceding two-year period. Hence, some relevant information for those items which have not had a regularly recurring pattern of buys is missing.

Despite the incomplete nature of most of the data, there was On-Order card data for about 4,000 items. For all these 4,000 items there is supplementary information from the Part List and the SNDS. For approximately 1,500 of these items there is also supplementary information from the Retention and Disposal Listing for 1961. In addition, we were able to obtain the relevant information for an additional 1,000 obsolete items from the Douglas Aircraft Company at El Segundo, California.

Information Contained in the Data

The following list contains the descriptive and quantitative variables available for the study, their limitations, and their source documents. The variables are randomly ordered with no particular thought to the relative importance of any one of them.

Federal Stock Number: (FSN) A 17 digit alphanumeric interservice part number by which all items are classified in all the catalogs used for the study. The third through the sixth digits are numerical and denote the Federal Supply Class. The fourteenth through the seventeenth digits are alphabetic and denote the Technical Supply Maintenance Code, the last three digits of which represent the manufacturer. The other digits are not significant for this study. A Federal Supply Class incorporates those items which support similar functions, although the particular items are dissimilar, e.g., 4120 (Fire Fighting Equipment) includes axes, ladders, and firehouse carts. Some classes, however, are restricted to similar parts only, e.g., 6240 (Light Bulbs and Lamps).

Unit Price: The average price paid for a given item in a given contract. Both setup and manufacturing costs are imputed in the unit price for each separate contract. Hence the unit price varies according to changing labor and setup costs associated with the different quantities purchased at different points in time. The unit price is sometimes listed in the SNDS and the On-Order Cards; for the On-Order Card, several different prices may be noted. A single price always appears in the Retention and Disposal Listing and the Parts List Catalog. The choice of price for the Parts List is not clear.

Nomenclature: A descriptive name for a part, which is designed to group an item by its physical characteristics and/or function. For the study the groups may be overly definitive, i.e., shims, washers, and spacers may be sufficiently similar as to constitute a single group. All the catalogs and cards indicate the nomenclature of an item.

Source Code: A code denoting the origin of a part, e.g., internal manufacturer, interservice transfer, or commercial contractor is found both in the SNDS and the Parts List.

BUSANDA Change Code: A code found in the SNDS indicating the reason that an item was declared obsolete if preceded by a "Q".

Life-Span: The number of months between initial BUWEPS procurement of an item and the declaration of obsolescence. This number may be computed from the date of the BUWEPS contract (prefixed "NOAS") listed first on the On-Order Card. If the date is absent from the card, it may be obtained from a list of BUWEPS contracts.

Since the early On-Order Cards for many items are often missing, two problems arise. First, no NOAS contract number may appear on the card, indicating that the first date on the card is for a re-order subsequent to initial procurement. Second, an item with several applications will have had several initial procurements, i.e., NOAS-coded contracts, one for each application. Hence the first NOAS-coded contract noted on a card may not be the first for that item. With either problem, the life-span for an item will be understated. Items with the first problem may be flagged to indicate that the life-span is understated. Those with the second problem cannot be so differentiated.

Quantity on Hand: That quantity of each item remaining at all Navy Supply Depots at the declaration of obsolescence. This figure is not available from any source. One approximation for each item is the quantity on hand at the last annual running of the Retention and Disposal program. The program computes an optimal maximum retention quantity for each item. Any quantity in stock greater than this maximum is termed surplus. Provided that a surplus existed at the time of the running, the item will appear on the Retention and Disposal listing, giving the maximum retention quantity and the quantity on hand. The most recent running of the program took place about six months prior to the publications of the SNDS catalogs used for this study. Hence, inclusion in the study only of items with "on-hand" date would give a bias towards items prone to surplus and discount all quantities of items withdrawn subsequent to the program, but prior to the declaration of obsolescence. Inclusion of the balance of the items requires that a zero on-hand quantity must be assigned to those items, which modifies the bias to those items which either had a surplus or whose stocks were exhausted at the precise moment that the declaration of obsolescence took place. It is not possible to isolate those items whose stocks had been exhausted prior to the declaration, and were unable to satisfy issue requests.

Number of Purchases: The number of times that re-orders were made is noted on the On-Order Cards.

Total Purchases: The sum of all the purchases made is found by adding the entries on the On-Order Cards.

Maintenance Per Cent: The percentage of anticipated replacements during maintenance is available from the Parts List.

Overhaul Per Cent: The percentage of anticipated replacements during overhauls is also given on the Parts List.

Application Codes: A code giving either the end item, i.e., the plane, on which the item goes, or the assembly of which the item is a replaceable part. This code, found in the Parts List, gives a measure of the interchangeability of a part within one plane, or among many planes.

<u>Units Per Application</u>: The quantity of a particular item needed to perform the desired function is found in the Parts List. It is possible, although difficult, to search out the total number of items used in all applications for a particular plane or set of planes.

C. Results of Statistical Analysis

First the pattern of obsolescence was studied. By pattern of obsolescence, we mean the frequency distributions of individual items according to their length of time-to-obsolescence (or life span), i.e., how many items had short, medium, or long life span? Thus, we are interested in studying the pattern of obsolescence that characterizes groups of items. Accordingly, the data were sorted by Federal Stock Code (FSC) and Technical Supply Maintenance Code (TSMC).

For each of the samples thus formed, a histogram was made of a distribution of life spans. Although most of the histograms did not exhibit any meaningful pattern, some of them did indicate a unimodal type distribution. Figures 1-4 show a series of histograms which are typical of this latter group.

Those probability distributions that were appropriate to this type of data were studied in order to find if any could be fitted to the data. It was found that the failure distribution $f(t) = a\, b\, e^{bt-a(e^{bt}-1)}$ (for discussion on the properties of this distribution, see Chapter V) closely approximated the patterns shown by the histograms. Trial values of the parameters a and b were tried out and a chi square test performed to indicate whether the failure distribution fitted the distributions of life spans for particular sets of parameters. It was found that the distributions of life spans of the FSC-TSMC combinations 1560-ADGA, 1650-ADGA, 2810-PWAC and 2840-PWAC (Figures 1-4) were reasonably described by failure distributions. The parameters a=1, b=.0343 were best

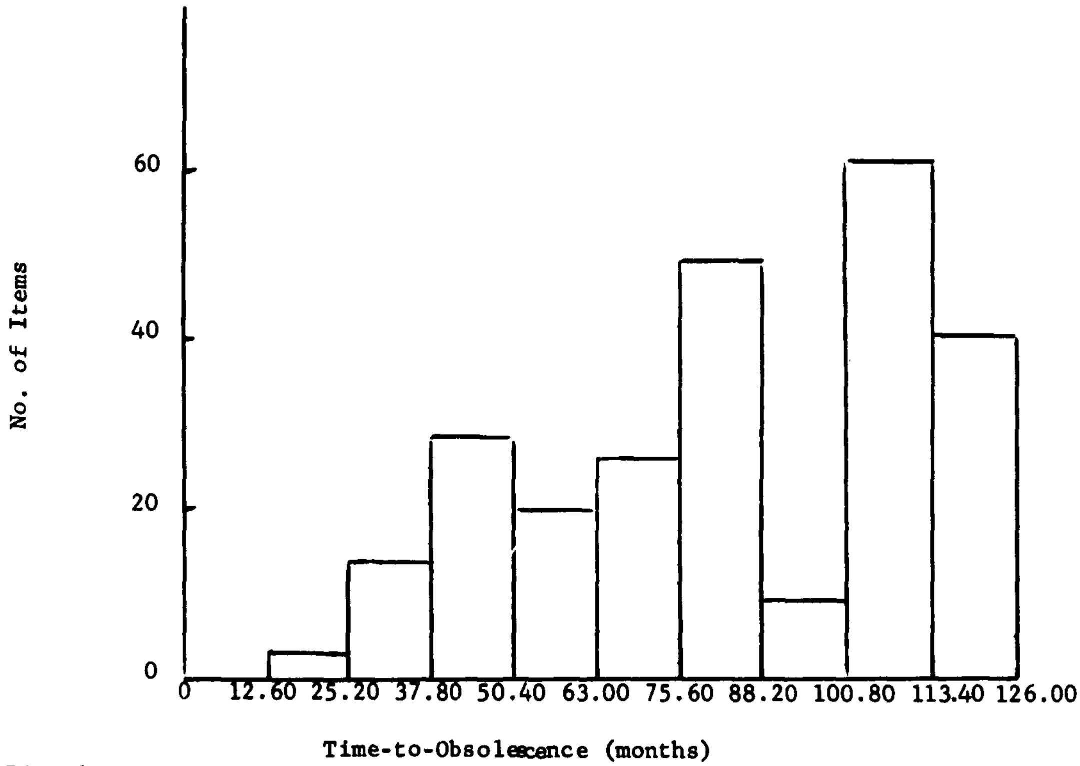

Fig. 1: Frequency Distribution of Time-to-Obsolescence for 1560-ADGA Sample*
*FSN 1560: Airframe structural components
TSMC ADGA: Douglas Aircraft Company

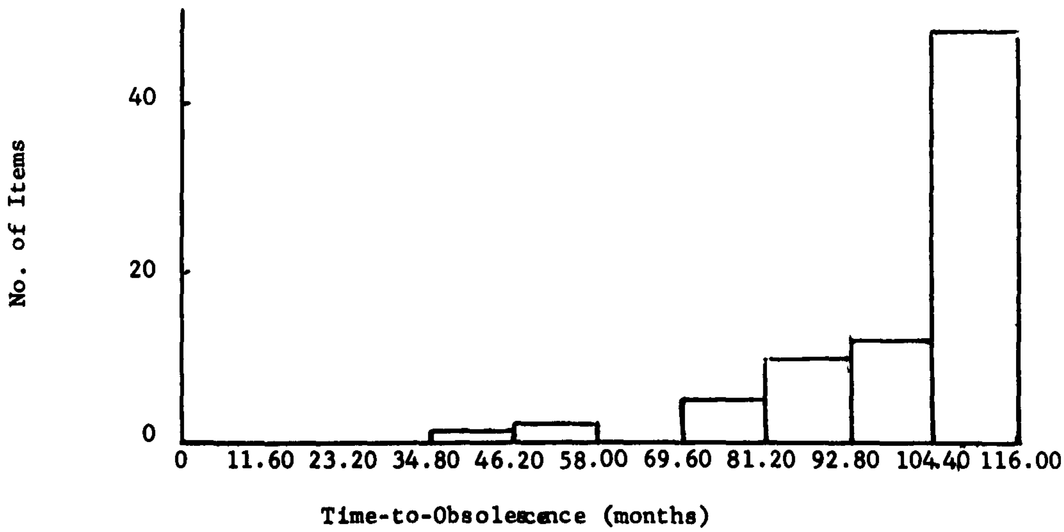

Fig. 2: Frequency Distribution of Time-to-Obsolescence for 1650-ADGA Sample*
*FSN 1650: Aircraft landing gear component

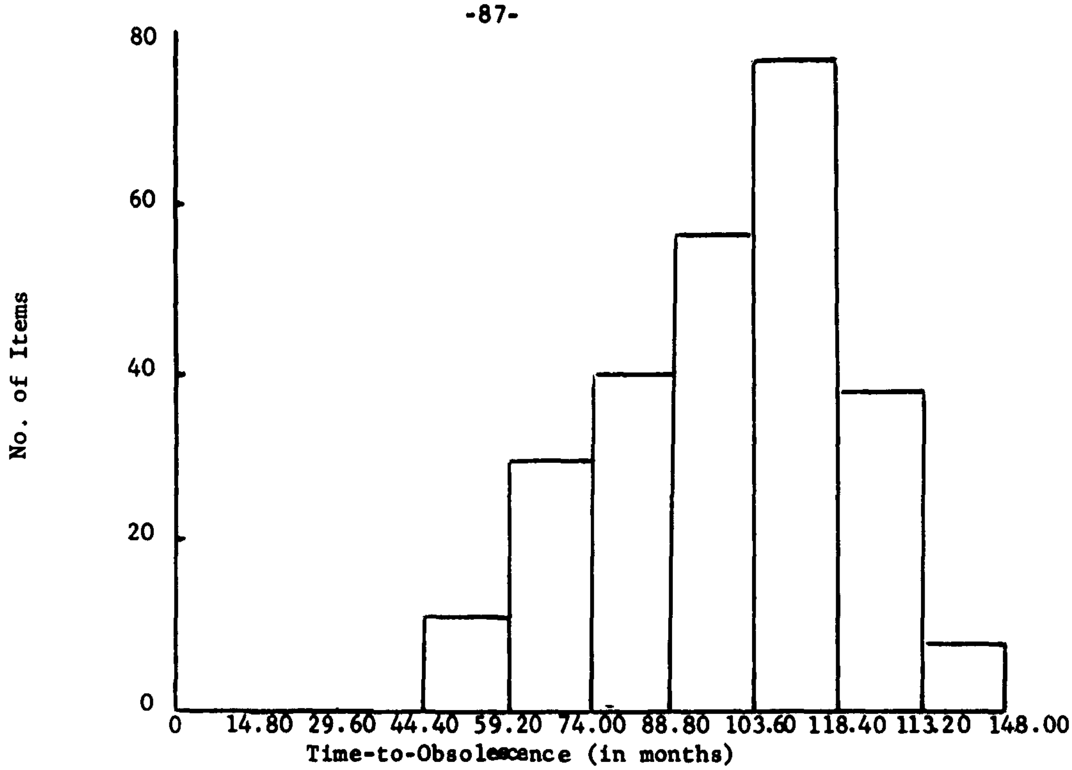

Fig. 3: Frequency Distribution of Time-to-Obsolescence for 2810-PWAC Sample*
*FSN 2810: Gasoline reciprocating engines, aircraft, & components
TSMC PWAL: Wright Aeronautical Corporation

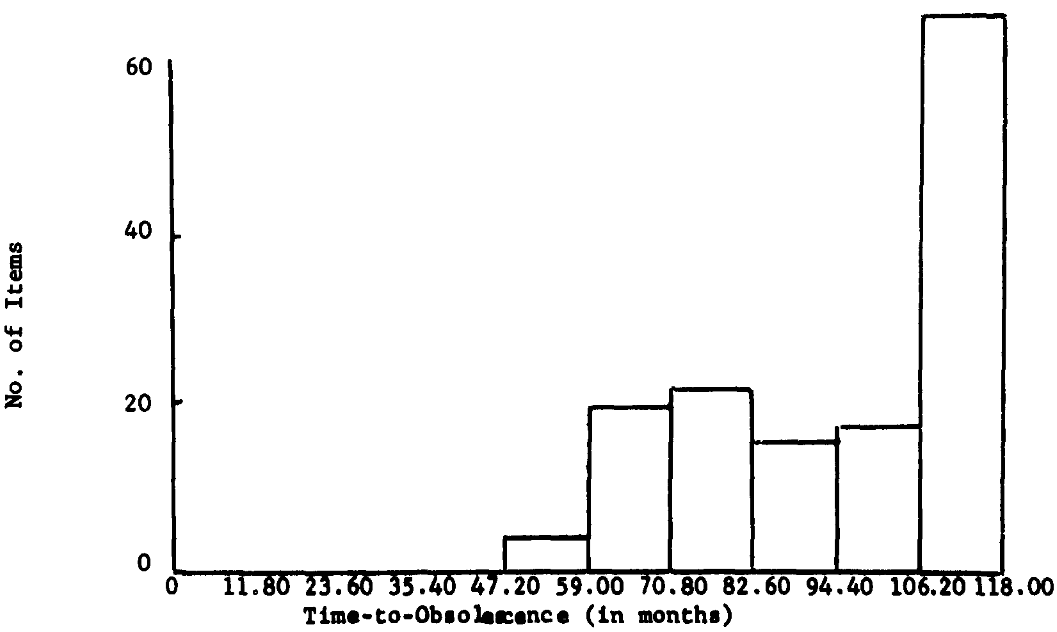

Fig. 4: Frequency Distribution of Time-to-Obsolescence for 2840-PWAC Sample*
*FSN 2840: Gas turbine and jet engine components

for ADGA distributions and a=1, b=.1429 best for PWAC distributions. Two particular values of life span, 116 and 118 from ADGA and PWAC distributions respectively, appeared to have abnormally high frequencies, so, were ignored when fitting distributions. The 1650-ADGA appeared to be short of its actual life span, but it resembled a truncated 1560-ADGA distribution so this was considered when fitting distribution. The 2840-PWAC distribution, although it was fitted, had frequencies occurring on a limited number of values of life span.

Using these four groups of data, a stepwise regression analysis was carried out using eight quantitative characteristics of each obsolescent item as the independent variables against the life span (y) of item as dependent variable. The eight variables are:

x_1 = unit price

X_2 = lead time

X_3 = total Quantity purchased

X_4 = total number of purchases

X_5 = number of applications

X_6 = number of units per application

X_7 = maintenance percent

X_8 = overhaul percent

Some of the data lacked quantitative information for lead time, total quantity purchased and number of purchases. These data were not used in the regression analysis. As a comparison, a regression analysis was also done on ADGA groups where frequencies of life span value 116 was excluded. Therefore, a total of six stepwise regression analyses were done. Since the F level to enter a variable was set at 4.5, no independent variable enters the step-wise regression analysis in two cases. That is to say, none of the eight quantitative variables were found to be statistically related to items life span. Results of the regression analyses are:

1) 1560-ADGA Group

a. Including items with life span of 116 months.

$$y = 88.92314 + 0.00643X - 3.66734X_2 + 4.17063X_3 + 2.95459X_4$$

$$(0.00272) \quad (1.14354) \quad (0.90705) \quad (1.29479)$$

Standard error of estimate = 24.7458

$R = 0.453$

b. Excluding items with life span of 116 months.

$$y = 88.41669 + 0.00729X_1 - 4.19640X_2 + 3.92748X_3$$

$$(0.00265) \quad (1.12390) \quad (0.92716)$$

Standard error of estimate = 23.1396

$R = 0.537$

2) 1650-ADGA Group

a. Including items with life span of 116 months.

$$y = 92.67528 + 5.65230X_4 - 2.20044X_5$$

$$(1.26133) \quad (0.85662)$$

Standard error of estimate = 15.4619

$R = 0.687$

excluded. Therefore, a total of six stepwise regression analyses were done. Since the F level to enter a variable was set at 4.5, no independent variable enters the step-wise regression analysis in two cases. That is to say, none of the eight quantitative variables were found to be statistically related to items life span. Results of the regression analyses are:

1) 1560-ADGA Group

a. Including items with life span of 116 months.

$$y = 88.92314 + 0.00643X - 3.66734X_2 + 4.17063X_3 + 2.95459X_4$$

$$(0.00272) \quad (1.14354) \quad (0.90705) \quad (1.29479)$$

Standard error of estimate = 24.7458

$R = 0.453$

b. Excluding items with life span of 116 months.

$$y = 88.41669 + 0.00729X_1 - 4.19640X_2 + 3.92748X_3$$

$$(0.00265) \quad (1.12390) \quad (0.92716)$$

Standard error of estimate = 23.1396

$R = 0.537$

2) 1650-ADGA Group

a. Including items with life span of 116 months.

$$y = 92.67528 + 5.65230X_4 - 2.20044X_5$$

$$(1.26133) \quad (0.85662)$$

Standard error of estimate = 15.4619

$R = 0.687$

b. Excluding items with life span of 116 months.

No independent variable entered the regression analysis.

3) 2810-PWAC Group

No independent variable was found to be significant.

4) 2840-PWAC Group

$$y = 104.28982 - \underset{(0.47045)}{1.27827X_2} - \underset{(53.05405)}{139.98844X_3}$$

Standard error of estimate = 14.2427

R = 0.466

The number in parenthesis and directly below each regression coefficient is the standard deviation of the estimate of coefficient.

From the above analysis, the following tentative conclusions may be drawn:

1. For airframe structural components, an item with higher unit value tends to have a longer life span. Perhaps there is some reluctance on the part of the inventory clerks to declare these high valued items as obsolescent items.

2. It appears that the longer the lead time the more likely that obsolescence will set in. This was the case for both airframe structural parts and engine parts. One conjecture is that the long lead time may mean a less flexible program. Since such a program is not desirable, from the standpoint of an efficient supply system, there is a tendency to eliminate the item concerned from the system.

3. In two cases, the regression analysis indicates that the total quantity purchased during an item's life span is positively correlated

to the life span itself. However, a plausible explanation is that if an item has a long life span, it is more likely that more purchases will be made.

As we consider how declarations of obsolescence have been made heretofore we can gain some appreciation of the reasons for the relatively weak dependencies of time-to-obsolescence upon such data as have been available.

There are, basically, two grounds for declaration of obsolescence of line items in Naval inventory. One is that on strategic grounds a weapons system is declared to be obsolete after some point in time. That is, it is declared that from some date on this system will be out of use. The other is that a stock control clerk in a supply-demand control point, following some rule of thumb as well as his time permits, and his assiduousness compels, notes absence of demand for items over a period of time, and makes a declaration of obsolescence if periods of sufficient length elapse during which demand is sufficiently low to satisfy the rule of thumb.

So far as the first ground for declaration of obsolescence is concerned, this is less a basis for termination of the system as a collection of line items in Naval inventory than a basis for the downgrading of the system in importance in the Navy's arsenal and its replacement by some other system or systems. This decision frequently commits the Navy to another decision, which is that no further procurements shall be made in support of the older system.

Conceivably, the application of models of the sorts developed in Chapters IV & VI will lead to a reorganization of the structure of these decisions, and therefore, to the entire process of obsolescence generation.

If a system has been downgraded it will continue in use for some period, but at a lower use rate. This, in turn, will reduce demand for spares items, insofar as the demand is due to wearout. The Bayesian feature of these models will then generate higher and higher probabilities that demand is running at lower levels. With known costs of disposal and of inventory operation, information can be fed to high level echelons at which final decisions about the obsolescence of a system can be made definitively.

When the matter at issue is the simpler case of a decision that an item can be declared obsolete by a stock control clerk because of low demand, this entire decision can be handled within the structure of the model, on a more thorough and reliable basis. Once again, the costs of carrying the item on inventory, and disposal costs, as well as stockout costs, would be taken into account in using the model.

This application of the models developed in this study to generate obsolescence decisions, along with the installation of appropriate data processing procedures of the sort sketched out in an accompanying proposal, would lead to a number of desirable results: sharper definition of obsolescence determination, less confusion in record keeping, an improvement in the quality of data about inventory line items. As a consequence, there would be developed a body of data which would allow analysts to arrive at reliable and meaningful conclusions as to the dependencies of time-to-obsolescence upon such data as characteristics of the item, its application, and the purchase contracts which embody the orders.

CHAPTER VIII
SUMMARY AND CONCLUSIONS

This chapter consists of a summary of the work which has been carried out under this contract, and some recommendations for further work which are spelled out in detail in accompanying proposals. In the course of this work an attempt was made to unearth meaningful empirical relationships between obsolescence and engineering, administrative and program data relating to Naval Aviation parts which are within the cognizance of the U.S. Naval Aviation Supply Office. In addition, a series of models was developed and explored to varying degrees which embody more and more sophisticated notions of just what obsolescence is operationally, and how it can be revealed in a useful fashion.

It has been found that by virtue of the nature of inventory administration procedures which are currently in use at ASO, and which are probably typical of procedures carried out at other Supply-Demand-Control Points, obsolescence is recorded in a fashion which is quite unpredictable, and which does not lend itself to meaningful statistical analysis. Recording of obsolescence, as well as maintenance of records as to dates, amounts and prices of purchases of items, is incomplete and quite dependent on such variable matters as work loads, assiduousness of clerical staff and their supervisors, availability of computing and computer programming facilities, etc. Moreover, it is our impression that these matters, particularly those relating to computing facilities, are serious bottlenecks.

Finally, even if these unplanned problems were cleared away, there remains the fundamental difficulty that the inventory control procedures which comprise the ideal, the plan according to which ASO tries to operate, represent a pastiche of notions, some inconsistent with each other, others redundant, with no real attempt to face a set of system-wide objectives. These procedures are described in Appendix C to this report. A comparison of this appendix with Chapter VIII of Whitin [13], which was published in 1957, is most illuminating. The chapter referred to contains a description of the Navy Inventory control system which was in effect prior to the publication of that book, with some diagnosis and recommendation for change.

For some time before and since the publication of Whitin's book BuSandA has been supporting research aimed at improvement of Navy Inventory procedures. However, it has appeared to those of us who spent a number of weeks at ASO and more weeks subsequently digesting what we had learned there, that thus far the effect of this research has been relatively small because results have been fed into existing organizations in small amounts, on tentative bases. Recommended procedures have been modified so substantially in order to accommodate existing organizational structures, ways of doing things and prejudices, as to be difficult to recognize. Moreover, there has been a tendency to take the position that what is good in one environment is good in isolation. Hence, a collage of measures, techniques and procedures has been assembled from widely scattered sources and these comprise the system.

As a consequence of the present study several models have been developed which represent successively better <u>global</u> approaches to the problem of managing a large complex inventory. The more sophisticated of these models enables the skilled inventory manager to take ancillary information, or what the Navy refers to as "program" data, into account in a systematic fashion. Specifically, if on budgetary or strategic grounds a particular weapons system is expected to become less significant in the activities of the Atlantic Fleet, and an aircraft type will be flying fewer hours, then *a priori* distributions of the probability of obsolescence, or *a priori* probabilities of being in one or another state of demand can be altered to reflect that expectation. Moreover, these models are flexible as to lead time, and other inventory parameters.

It is true that these models make certain rather stringent demands of the inventory manager which cannot be solved simply by improving the quality of data handling. All of these demands are in the area of acquiring better understanding of the cost nature of various conditions of the inventory. How much does it really cost the Navy to be out of stock of a particular part? How much does it really cost to carry such and such an item in inventory, to order it, etc.? These questions should be answered, somehow or other. However, even if the first-cut answers are semi-educated guesses, they will enable the models to be operated, and in fact these answers could be checked in terms of the resultant inventory policies which would be generated.

What is suggested, then, is that these models can eventually serve as inventory control procedures. Indeed, proposals for further study and for installation accompany this report. However, what is perhaps more significant immediately is that these models could serve as measuring instruments for the development of measures of some of the more fugitive inventory parameters of which theoreticians speak so casually, such as stockout cost, setup cost, etc. Thus, a sensible procedure which might be adopted could involve four phases (discussed in detail in the accompanying proposal for further work):

1. Set up and begin operation of a computer system designed to look up appropriate control policies for separate items. These policies would be generated elsewhere according to some model.
2. At the same time, continue exploration of models. Eventually, tables should be generated from the appropriate model or models which would be available for lookup in inventory control.
3. Load into the machine system, as they are developed, tables based on successively more satisfactory models. Running parallel to the existing inventory control system, run this new system and compare results. As confidence in these tables developes, studies are begun, using these models, which would lead to measures of inventory system parameters consistent with current policies, (i.e. policies in use before any of these changes have begun). This procedure, iterated and played against inventory managers and those responsible for high level logistical policy, would lead, eventually, to acceptable measures of such parameters as setup cost, lead time, stockout cost, etc. In addition, during this phase, studies

could be carried out which would reveal, in detail, the amount of savings in physical inventory costs which would accrue from the installation of the system, as well as the improvement in quality of inventory data. Note that up to this time the normal operations of the SDCP have not been interfered with. All the work described in these three phases has been carried on outside the current operation.

4. Once satisfaction has been achieved with respect to the utility of the new system, it could be phased in gradually, while the old one is phased out. Conceivably, with low demand, high value items, or with items with very high stockout costs or other special problems, phase-in might be a much slower process. Indeed, it might be desirable that some items never be completely controlled by mechanized procedures, but be constantly subject to review by highly skilled inventory specialists. However, for the bulk of the 400,000 items within the cognizance of ASO, as well, probably, as the remainder of the 1.2 million items handled by other SDCP's for the Navy, this sort of procedure would probably turn out to be a most economical and satisfactory one.

Although it will not be gone into detail here conservative preliminary estimates indicate that inventory administration costs (i.e. costs of maintaining files, ordering, disposing, etc.) for the 400,000 items handled by ASO, could probably be reduced by a substantial amount each year. This does not take into account the actual physical inventory savings due to changes in amounts of inventory held, changes in numbers of stockouts, disposals, orders, etc. What is being asserted here is that the costs of placing exactly the same orders, carrying exactly the same amounts of all items, disposing of the same amounts, etc. at the same times, would be much less than they now are. Savings resulting from changing these policies would, in all likelihood, be much more substantial, but their magnitude would begin to be revealed only during the third phase of the four phase procedure laid out above.

APPENDIX A

POLICIES RESULTING FROM DYNAMIC PROCUREMENT - DISPOSAL MODEL RUNS

Before presenting the results of the computer runs, we shall first describe how the values of parameters, which characterize the demand and obsolescence distributions and cost functions, are varied in each of the runs.

Each run is identified by a three-digit number. The first digit identifies one of the five different sets of assumptions regarding the demand and obsolescence distributions. The last two digits identify a different cost combination. These identification codes are indicated in Tables 1-A and 2-A.

Policies resulting from each computer run are tabulated in the remainder of this appendix. Note that for each run, we assumed an inventory process with a finite horizon consisting of 10 periods.

TABLE 1-A

Variations in Demand and

Obsolescence Distributions

Run #	Demand Distribution	Obsolescence Distribution
1XX	Poisson demand with mean of 30; demand rate not changing through time	Exponential distribution with obsolescence rate of 10%.
2XX	Poisson demand with decreasing mean demand rate through time as follows: Time / Mean demand 1,2 / 40 3,4 / 35 5,6 / 30 7,8 / 25 9,10 / 20	Same as in 1XX runs.
3XX	Uniform demand with mean of 30; demand rate not changing through time	Same as in 1XX runs.
4XX	Same as in 3XX runs	Uniform distributions: 10% obsolescence rate.
5XX	Uniform distribution decreasing mean demand rate as in 2XX runs	Same as in 4XX runs.

TABLE 2-A

Variations in Cost Parameters

Run #	Disposal*,** Cost	Fixed Ordering* Cost	Storage*,** Cost	Out-of-stock*,** Cost
X 01	0-2 Z	10	0.1 Z	5 Z
X 02	0-2 Z	10	0.1 Z	30 Z
X 03	0-2 Z	40	0.05 Z	10 Z
X 04	0-2 Z	40	0.05 Z	30 Z
X 05	0-2 Z	40	0.1 Z	30 Z
X 06	15-2 Z	10	0.1 Z	5 Z
X 07	15-2 Z	10	0.2 Z	5 Z
X 08	15-2 Z	10	0.1 Z	30 Z
X 09	15-2 Z	40	0.1 Z	5 Z
X 10	15-2 Z	40	0.1 Z	30 Z
X 11	0-2 Z	40	0.02 Z	10 Z
X 12	0-2 Z	40	0.02 Z	30 Z

* All costs are expressed as multiples of unit price of the stock. (i.e., unit price = 1)

** Z = amount to be disposed, stored or to which a penalty cost is charged.

TABLE 3-A

Policies Resulting from Model Runs*

Run 101

Period**	H_1	K_1	K_2	H_2
10	27	34	43	44
9	29	38	77	78
8	29	38	108	109
7	29	38	137	138
6	29	38	165	166
5	29	38	192	193
4	29	38	216	217
3	29	38	238	239
2	29	38	256	257
1	29	38	261	262

Run 102

Period**	H_1	K_1	K_2	H_2
10	34	39	46	47
9	35	42	81	82
8	35	42	112	113
7	35	42	142	143
6	35	42	170	171
5	35	42	196	197
4	35	42	221	222
3	35	42	242	243
2	35	42	260	261
1	35	42	265	266

Run 103

Period**	H_1	K_1	K_2	H_2
10	27	37	47	48
9	29	68	82	83
8	27	70	115	116
7	28	70	146	147
6	27	70	176	177
5	28	70	206	207
4	28	70	235	236
3	28	70	263	264
2	28	70	290	291
1	28	70	-	-

Run 104

Period**	H_1	K_1	K_2	H_2
10	31	39	47	48
9	33	70	84	85
8	32	73	117	118
7	32	74	148	149
6	32	73	179	180
5	32	74	208	209
4	32	73	235	236
3	32	74	265	266
2	32	73	292	293
1	32	74	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 105

Period**	H_1	K_1	K_2	H_2
10	31	39	46	47
9	32	70	82	83
8	32	73	114	115
7	32	73	145	146
6	32	73	174	175
5	32	73	202	203
4	32	73	229	230
3	32	73	254	255
2	32	73	276	277
1*	32(?)	73(?)	***	***

Run 106

Period**	H_1	K_1	K_2	H_2
10	25	32	41	199
9	28	37	73	164
8	28	37	104	173
7	28	37	133	192
6	28	37	161	215
5	28	37	188	240
4	28	37	213	266
3	28	37	236	290
2	28	37	-	-
1	28	37	-	-

Run 107

Period**	H_1	K_1	K_2	H_2
10	25	32	39	118
9	28	37	71	117
8	28	37	100	137
7	28	37	127	161
6	28	37	153	186
5	28	37	174	211
4	28	37	189	233
3	28	37	189	250
2	28	37	189	258
1	28	37	189	259

Run 108

Period**	H_1	K_1	K_2	H_2
10	33	38	45	203
9	35	42	79	169
8	35	42	110	178
7	35	42	139	198
6	35	42	167	221
5	35	42	194	246
4	35	42	219	271
3	35	42	242	296
2	35	42	-	-
1	35	42	-	-

---

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

**Real time periods; i.e. period n + 1 is later than period n.

***Through an operator error, the full output for this period was not printed. H_1 and K_1 values were inferred from the partial output, but the H_2 and K_2 values could not be so inferred.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 109

Period**	H_1	K_1	K_2	H_2
10	23	35	44	202
9	25	65	77	167
8	22	67	108	176
7	24	67	138	196
6	23	67	167	220
5	24	67	195	246
4	23	67	222	272
3	24	67	248	298
2	23	67	-	-
1	24	67	-	-

Run 110

Period**	H_1	K_1	K_2	H_2
10	31	39	46	203
9	32	70	80	170
8	32	73	112	180
7	32	73	142	200
6	32	73	171	224
5	32	73	199	250
3	32	73	227	277
3	32	73	-	-
2	32	73	-	-
1	32	73	-	-

Run 111

Period**	H_1	K_1	K_2	H_2
10	27	38	48	49
9	29	68	85	86
8	27	95	119	120
7	28	71	151	152
6	28	71	183	184
5	28	71	214	215
4	28	71	244	245
3	28	71	273	274
2	28	71	-	-
1	28	71	-	-

Run 112

Period**	H_1	K_1	K_2	H_2
10	31	39	48	49
9	33	71	86	87
8	32	97	121	122
7	33	74	153	154
6	32	74	185	186
5	32	74	216	217
4	32	74	246	247
3	32	74	275	276
2	32	74	-	-
1	32	74	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

**Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 201

Period**	H_1	K_1	K_2	H_2
10	17	23	31	32
9	18	26	54	55
8	23	32	81	82
7	23	32	105	106
6	29	38	134	135
5	29	38	162	163
4	34	43	193	194
3	34	43	222	223
2	39	49	253	254
1	39	49	281	282

Run 202

Period**	H_1	K_1	K_2	H_2
10	23	27	33	34
9	24	30	57	58
8	29	36	84	85
7	29	36	109	110
6	35	42	138	139
5	35	42	165	166
4	41	48	196	197
3	41	48	225	226
2	46	54	257	258
1	46	54	284	285

Run 203

Period**	H_1	K_1	K_2	H_2
10	17	26	34	35
9	19	47	58	59
8	23	70	87	88
7	23	61	113	114
6	28	65	145	146
5	28	90	175	176
4	33	76	209	210
3	33	81	243	244
2	38	86	281	282
1	37	92	-	-

Run 204

Period**	H_1	K_1	K_2	H_2
10	20	28	35	36
9	22	49	60	61
8	27	72	88	89
7	27	64	115	116
6	32	68	146	147
5	32	74	177	178
4	38	79	211	212
3	37	85	245	246
2	43	90	282	283
1	43	95	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy K_1 - H
$H \geq H_2$	Dispose H - K_2

**Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 205

Period**	H_1	K_1	K_2	H_2
10	20	28	34	35
9	22	49	58	59
8	27	71	86	87
7	27	63	112	113
6	32	68	142	143
5	32	74	171	172
4	38	78	204	205
3	37	84	235	236
2	43	89	269	270
1	42	95	-	-

Run 206

Period**	H_1	K_1	K_2	H_2
10	15	21	28	185
9	18	26	51	140
8	23	32	77	145
7	23	31	101	160
6	28	37	130	183
5	28	37	157	209
4	33	43	189	240
3	33	43	219	271
2	38	48	-	-
1	38	48	-	-

Run 207

Period**	H_1	K_1	K_2	H_2
10	15	21	27	105
9	17	25	49	94
8	23	31	73	109
7	23	31	96	127
6	28	37	123	155
5	28	37	147	181
4	33	42	173	210
3	33	42	192	238
2	38	48	206	265
1	38	48	221	285

Run 208

Period**	H_1	K_1	K_2	H_2
10	22	27	32	189
9	24	30	56	145
8	29	36	82	150
7	29	36	106	165
6	35	42	135	188
5	35	42	162	214
4	41	48	194	245
3	41	48	223	276
2	46	54	-	-
1	46	54	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

**Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 209

Period**	H_1	K_1	K_2	H_2
10	13	24	32	189
9	16	44	54	143
8	19	66	80	148
7	18	58	106	164
6	24	62	135	188
5	23	68	164	214
4	29	72	197	246
3	28	78	228	278
2	33	83	-	-
1	32	88	-	-

Run 210

Period**	H_1	K_1	K_2	H_2
10	20	27	33	190
9	22	48	57	145
8	27	71	84	151
7	26	63	109	167
6	32	68	139	192
5	32	74	168	218
4	38	78	200	250
3	37	84	232	282
2	43	89	-	-
1	42	95	-	-

Run 211

Period**	H_1	K_1	K_2	H_2
10	17	26	35	36
9	19	47	61	62
8	23	70	90	91
7	23	61	118	119
6	28	66	150	151
5	28	91	181	182
4	33	76	217	218
3	33	82	252	253
2	38	87	292	293
1	38	92	-	-

Run 212

Period**	H_1	K_1	K_2	H_2
10	20	28	35	36
9	22	49	62	63
8	27	72	91	92
7	27	64	119	120
6	33	68	152	153
5	32	95	183	184
4	38	79	219	220
3	37	85	254	255
2	43	90	294	295
1	43	96	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

**Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 301

Period**	H_1	K_1	K_2	H_2
10	30	44	60	61
9	41	56	110	111
8	41	57	147	148
7	41	57	174	175
6	41	57	199	200
5	41	57	220	221
4	41	57	237	238
3	41	57	251	252
2	41	57	262	263
1	41	57	270	271

Run 302

Period**	H_1	K_1	K_2	H_2
10	50	57	60	61
9	53	60	116	117
8	53	60	156	157
7	53	60	182	183
6	53	60	206	207
5	53	60	226	227
4	53	60	243	244
3	53	60	257	258
2	53	60	267	268
1	53	60	274	275

Run 303

Period**	H_1	K_1	K_2	H_2
10	33	53	60	61
9	39	60	117	118
8	39	81	162	163
7	39	101	199	200
6	39	102	228	229
5	39	102	256	257
4	39	102	281	282
3	39	102	-	-
2	39	102	-	-
1	39	102	-	-

Run 304

Period**	H_1	K_1	K_2	H_2
10	45	58	60	61
9	48	62	118	119
8	48	82	167	168
7	48	109	204	205
6	48	110	233	234
5	48	110	260	261
4	48	110	285	286
3	48	110	-	-
2	48	110	-	-
1	48	110	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy K_1 - H
$H \geq H_2$	Dispose H - K_2

**Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 305

Period**	H_1	K_1	K_2	H_2
10	45	58	60	61
9	47	60	116	117
8	47	71	162	163
7	47	94	191	192
6	47	109	216	217
5	47	109	239	240
4	47	109	260	261
3	47	109	277	278
2	47	109	291	292
1	47	109	-	-

Run 306

Period**	H_1	K_1	K_2	H_2
10	28	42	59	207
9	40	55	105	194
8	40	56	137	213
7	40	56	164	236
6	40	56	189	260
5	40	56	210	283
4	40	56	-	-
3	40	56	-	-
2	40	56	-	-
1	40	56	-	-

Run 307

Period**	H_1	K_1	K_2	H_2
10	28	41	58	133
9	39	54	99	149
8	40	55	124	175
7	40	55	147	198
6	40	55	165	219
5	40	55	181	236
4	40	55	192	250
3	40	55	198	261
2	40	55	201	267
1	40	55	202	271

Run 308

Period**	H_1	K_1	K_2	H_2
10	50	56	60	208
9	53	60	114	200
8	53	60	146	222
7	53	60	172	244
6	53	60	196	268
5	53	60	218	291
4	53	60	-	-
3	53	60	-	-
2	53	60	-	-
1	53	60	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 309

Period**	H_1	K_1	K_2	H_2
10	19	46	60	208
9	31	60	109	197
8	29	67	143	217
7	30	89	171	242
6	30	91	197	268
5	30	91	221	293
4	30	91	-	-
3	30	91	-	-
2	30	91	-	-
1	30	91	-	-

Run 310

Period**	H_1	K_1	K_2	H_2
10	44	57	60	208
9	47	60	114	200
8	47	71	153	228
7	47	93	180	251
6	47	109	207	277
5	47	109	-	-
4	47	109	-	-
3	47	109	-	-
2	47	109	-	-
1	47	109	-	-

Run 311

Period**	H_1	K_1	K_2	H_2
10	32	52	-	-
9	39	60	-	-
8	39	85	-	-
7	39	102	-	-
6	39	103	-	-
5	39	103	-	-
4	39	103	-	-
3	39	103	-	-
2	39	103	-	-
1	39	103	-	-

Run 312

Period**	H_1	K_1	K_2	H_2
10	45	57	-	-
9	48	62	-	-
8	48	87	-	-
7	48	110	-	-
6	48	111	-	-
5	48	110	-	-
4	48	110	-	-
3	48	110	-	-
2	48	110	-	-
1	48	110	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy K_1 - H
$H \geq H_2$	Dispose H - K_2

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 401

Period**	H_1	K_1	K_2	H_2
10	30	44	60	61
9	36	51	107	108
8	39	54	137	138
7	40	56	160	161
6	41	57	182	183
5	42	57	204	205
4	42	58	223	224
3	42	58	243	244
2	43	59	261	262
1	43	59	278	279

Run 402

Period**	H_1	K_1	K_2	H_2
10	50	57	60	61
9	52	59	114	115
8	52	59	145	146
7	53	60	167	168
6	53	60	189	190
5	53	60	209	210
4	53	60	229	230
3	53	60	248	249
2	53	60	265	266
1	53	60	282	283

Run 403

Period**	H_1	K_1	K_2	H_2
10	33	53	60	61
9	37	58	115	116
8	38	60	157	158
7	38	60	187	188
6	39	96	214	215
5	41	104	241	242
4	41	107	266	267
3	42	110	291	292
2	42	112	-	-
1	42	114	-	-

Run 404

Period**	H_1	K_1	K_2	H_2
10	45	58	60	61
9	47	60	117	118
8	47	60	162	163
7	47	60	191	192
6	48	97	218	219
5	49	111	245	246
4	49	113	270	271
3	49	114	295	296
2	50	116	-	-
1	50	117	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 405

Period**	H_1	K_1	K_2	H_2
10	45	58	60	61
9	46	60	115	116
8	47	60	153	154
7	47	60	174	175
6	47	73	200	201
5	48	109	222	223
4	49	112	244	245
3	49	113	265	266
2	49	114	285	286
1	49	115	-	-

Run 406

Period**	H_1	K_1	K_2	H_2
10	28	42	59	207
9	35	50	102	209
8	38	53	129	219
7	40	55	153	233
6	40	56	175	251
5	41	57	197	270
4	42	58	217	289
3	42	58	-	-
2	42	58	-	-
1	42	59	-	-

Run 407

Period**	H_1	K_1	K_2	H_2
10	28	41	58	133
9	34	49	95	155
8	37	52	113	171
7	39	54	135	190
6	40	55	153	207
5	40	56	168	223
4	41	57	183	238
3	41	57	197	252
2	41	57	209	265
1	42	58	220	277

Run 408

Period**	H_1	K_1	K_2	H_2
10	50	56	60	208
9	52	58	112	216
8	52	59	137	228
7	53	60	161	241
6	53	60	182	259
5	53	60	204	278
4	53	60	223	296
3	53	60	-	-
2	53	60	-	-
1	53	60	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 409

Period**	H_1	K_1	K_2	H_2
10	19	46	60	208
9	26	54	106	212
8	28	58	136	224
7	29	60	160	239
6	30	72	184	259
5	32	94	207	279
4	32	98	229	300
3	33	101	-	-
2	33	104	-	-
1	34	106	-	-

Run 410

Period**	H_1	K_1	K_2	H_2
10	44	57	60	208
9	46	59	113	217
8	47	60	145	234
7	47	60	169	248
6	47	72	193	268
5	48	109	216	288
4	49	112	-	-
3	49	113	-	-
2	49	114	-	-
1	49	115	-	-

Run 411

Period**	H_1	K_1	K_2	H_2
10	32	52	-	-
9	36	58	-	-
8	37	60	-	-
7	38	60	-	-
6	39	100	-	-
5	41	105	-	-
4	41	109	-	-
3	42	111	-	-
2	42	114	-	-
1	43	116	-	-

Run 412

Period**	H_1	K_1	K_2	H_2
10	45	57	-	-
9	46	60	-	-
8	47	60	-	-
7	47	60	-	-
6	48	108	-	-
5	49	112	-	-
4	49	114	-	-
3	49	115	-	-
2	50	117	-	-
1	50	119	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy K_1 - H
$H \geq H_2$	Dispose H - K_2

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 501

Period**	H_1	K_1	K_2	H_2
10	19	30	40	41
9	23	35	72	73
8	31	45	100	101
7	33	47	123	124
6	41	56	151	152
5	42	57	177	178
4	50	67	208	209
3	50	68	237	238
2	59	77	271	272
1	59	78	-	-

Run 502

Period**	H_1	K_1	K_2	H_2
10	33	38	40	41
9	34	39	77	78
8	43	49	106	107
7	44	50	128	129
6	53	60	155	156
5	53	60	181	182
4	63	70	213	214
3	63	70	241	242
2	72	80	275	276
1	72	80	-	-

Run 503

Period**	H_1	K_1	K_2	H_2
10	20	37	40	41
9	22	40	78	79
8	31	50	114	115
7	31	57	142	143
6	40	88	176	177
5	41	103	206	207
4	49	115	244	245
3	50	125	278	279
2	58	136	-	-
1	59	146	-	-

Run 504

Period**	H_1	K_1	K_2	H_2
10	29	39	40	41
9	29	40	79	80
8	39	50	118	119
7	39	57	145	146
6	48	91	179	180
5	49	110	209	210
4	58	121	247	248
3	58	131	282	283
2	67	142	-	-
1	68	152	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 505

Period**	H_1	K_1	K_2	H_2
10	28	39	40	41
9	29	40	77	78
8	38	50	113	114
7	38	50	134	135
6	48	71	165	166
5	48	109	194	195
4	57	120	227	228
3	58	130	257	258
2	67	140	294	295
1	67	150	-	-

Run 506

Period**	H_1	K_1	K_2	H_2
10	16	28	39	186
9	21	33	68	172
8	30	44	93	179
7	32	46	116	191
6	41	56	143	214
5	41	57	168	238
4	50	66	198	270
3	50	67	226	298
2	59	77	-	-
1	59	78	-	-

Run 507

Period**	H_1	K_1	K_2	H_2
10	16	27	39	113
9	21	33	63	120
8	30	43	83	135
7	31	45	102	152
6	40	55	127	177
5	40	56	147	198
4	49	65	172	226
3	49	66	194	250
2	58	75	220	279
1	58	76	-	-

Run 508

Period**	H_1	K_1	K_2	H_2
10	32	38	40	187
9	33	39	75	177
8	43	49	99	186
7	43	50	122	197
6	53	60	149	220
5	53	60	173	243
4	62	70	203	275
3	63	70	-	-
2	72	80	-	-
1	72	80	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 509

Period**	H_1	K_1	K_2	H_2
10	10	32	40	187
9	14	38	72	176
8	22	49	99	184
7	22	50	122	196
6	31	70	151	222
5	31	94	178	247
4	39	105	209	280
3	40	115	-	-
2	48	126	-	-
1	48	135	-	-

Run 510

Period**	H_1	K_1	K_2	H_2
10	28	39	40	187
9	29	40	76	178
8	38	50	107	191
7	38	50	129	203
6	48	70	157	229
5	48	109	185	253
4	57	120	216	287
3	58	130	-	-
2	67	140	-	-
1	67	150	-	-

Run 511

Period**	H_1	K_1	K_2	H_2
10	19	36	-	-
9	22	40	-	-
8	30	50	-	-
7	30	63	-	-
6	40	91	-	-
5	41	104	-	-
4	49	116	-	-
3	50	126	-	-
2	58	138	-	-
1	59	148	-	-

Run 512

Period**	H_1	K_1	K_2	H_2
10	29	39	40	41
9	30	40	80	81
8	39	50	122	123
7	49	63	156	157
6	48	96	192	193
5	49	111	227	228
4	58	122	268	269
3	59	132	-	-
2	67	143	-	-
1	68	153	-	-

*Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period n + 1 is later than period n.

APPENDIX B

FORTRAN PROGRAMS FOR INVENTORY MODELS DEVELOPED IN CHAPTER 4 AND 6

The listings of the FORTRAN programs used for calculating optimal policies in Chapters 4 and 6 are presented in this appendix. The program decks are available on request.

FORTRAN Program for Dynamic Procurement-Disposal Model in Chapter IV.

```
..JOB1
      PROGRAM XINVENT
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION MSI(4001),MMK(2001)
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
   75 CONTINUE
      WRITE OUTPUT TAPE 6,9876
 9876 FORMAT(1H1)
      READ INPUT TAPE 5,2001,NRUN,EPS
      WRITE OUTPUT TAPE 6,2001,NRUN,EPS
 2001 FORMAT(I10/F10.8)
   31 I2=1
      N1=0
      DO30 I=1,20
   30 ICOUNT(I)=0
      IFLAG=0
      READ INPUT TAPE5,2000,IFLAG
 2000 FORMAT(I10)
      CALL ASUBI
      READ INPUT TAPE 5,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST
      WRITE OUTPUT TAPE 6,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST
 2003 FORMAT(8I5)
    8 CALL DATA
      CALL THETA
      ISTRT=JPA+1-MAXI
      IEND=JPA+1
      KTRL=1
      DO 200 IP=ISTRT,IEND
      IT=JPA+1-IP
      GO TO (10,11),KTRL
   10 IF(IT-IREAL)12,11,11
   12 READ INPUT TAPE 5,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST
```

```
      WRITE OUTPUT TAPE 6,2003,IREAL,JK,JCST,JGIS,JCOT,JTHET,JRATE,JDIST
      IF(IREAL)13,14,13
   13 CALL DATA
      GO TO 10
   14 KTRL=2
   11 CONTINUE
      XMU(IP)=XMEAN
      SMALA=PA(IP)/(ASUM+PA(IP))
      ASUM=ASUM+PA(IP)
      IF(JK)15,16,15
   16 IF(JTHET)15,17,15
   15 CALL THETA
   17 CONTINUE
      IF(IP-ISTRT)105,106,105
  106 I1=2*K+1
  107 DO102 I=1,I1
      MSI(I)=K+1-I
      IF(MSI(I))102,104,104
  104 MMK(I)=I-1
  102 VMIN(I)=THET(I)
      IMAX=IT
      GO TO 67
  105 IH=K
      KUP1=2*IH+1
      KUP2=K+1
      DO 199 IHH=1,KUP1
      VMIN(IHH)=1.E+25
      DO 198 KK=1,KUP2
      KDUM=IHH+KK-1
      V=PHI(KK)/(1.+RATE)+THET(KDUM)
      IF (VMIN(IHH)-V)198,198,45
   45 VMIN(IHH)=V
      IS(IHH)=KK-1
  198 CONTINUE
  199 CONTINUE
      LLJ=1
      LLK=XMINOF(120,I1)
```

```
      IF(IFLAG)300,61,300
  300 N1=N1+1
      N2=1
      I2=I2
      DO310 J=1,I1
      GO TO(301,302,303),N2
  301 IF(IS(J)-MSI(J))310,304,310
  302 IF(IS(J)-MSI(J))310,310,306
  303 IF(IS(J)-IS(J+1))307,310,307
  304 N2=2
      I4=J-1
      IF(J-1)305,305,308
  305 IH1(I2)=0
      IK(I2)=0
      XM(I2)=0
      I2=I2+1
      GO TO 310
  306 N2=3
      I4=J
      ICOUNT(N1)=ICOUNT(N1)+1
      GO TO 308
  307 N2=1
      I4=J+1
      IF(J-I1)308,310,310
  308 IH1(I2)=MSI(I4)
      IK(I2)=IS(I4)
      XM(I2)=VMIN(I4)
      I2=I2+1
  310 CONTINUE
      GO TO 67
   61 WRITE OUTPUT TAPE6,1012
 1012 FORMAT(16H1   ORDER POLICY /56H     H=AMOUNT ON HAND AT THE END OF
     C PREVIOUS TIME PERIOD)
      WRITE OUTPUT TAPE 6,1013
 1013 FORMAT(53H     M(H)=EXPECTED COST OF POLICY FROM THIS PERIOD ON)
   64 WRITE OUTPUT TAPE 6,1014,(MSI(I),IS(I),VMIN(I),I=LLJ,LLK)
 1014 FORMAT(115H1          H           K           M(H)          H
```

```
   X          K              M(H)                H                K            M(H)/(3(2I
  C12,F14.2,1H )))
 63 IF(LLK-I1)66,67,67
 66 LLJ=LLK+1
    LLK=LLK+XMINOF(120,I1-LLK)
    IF(IFLAG)67,64,67
 67 IF(IT)70,74,70
 74 TEMX=0.
    DO 90 I=ISTRT,JPA
    IJ=JPA+1-I
    XI=I
 90 TEMX=TEMX+XMU(IJ)*PA(IJ)*XI
    IF(IFLAG)320,91,320
320 WRITE OUTPUT TAPE6,1012
    WRITE OUTPUT TAPE6,1013
    WRITE OUTPUT TAPE6,1018
1018 FORMAT(/54H     IF H IS GREATER THAN OR EQUAL TO H2, DISPOSE H-K2/
   X47H     IF H IS LESS THAN OR EQUAL TO H1, BUY K1-H//5X,6HPERIOD,3X
   X,2HH1,5X,2HK1,5X,4HM(H),5X,2HH2,5X,2HK2,5X,4HM(H)/)
    J1=1
    DO325 I=1,IMAX
    IT=IMAX+1-I
    J2=J1+2*(ICOUNT(I)-1)
    WRITE OUTPUT TAPE6,2010,IT,(IH1(J+1),IK(J+1),XM(J+1),IH1(J),IK(J),
   XXM(J),J=J1,J2,2)
2010 FORMAT(I9,(2(2I7,F9.2)/)/)
325 J1=J2+2
 91 WRITE OUTPUT TAPE6,1021,TEMX
1021 FORMAT(40H1 EXPECTED TOTAL DEMAND THROUGH LIFETIME F10.2)
    GO TO 75
 70 WRITE OUTPUT TAPE 6,1015,IT,SMALA
1015 FORMAT(25H1  BEGIN REAL TIME PERIOD I3,81H.  CONDITIONAL PROBABILI
   CTY OF OBSOLESCENCE OCCURRING AT THE END OF THIS PERIOD IS F9.6)
 94 KUP1=K+1
    KUP2=IR
    DO 197 KK=1,KUP1
    PHI(KK)=0.
```

```
      LLL=IS(4001)
      DO 196 IRR=LLL,KUP2
      IF(KK-IRR)51,50,50
   50 CALL PSIONE
      GO TO 52
   51 CALL PSITWO
   52 PHI(KK)=PHI(KK)+PRO(IRR)*PSI
  196 CONTINUE
  197 CONTINUE
      LLJ=1
      LLK=XMINOF(200,K+1)
      IF(IFLAG-1)195,195,96
  195 WRITE OUTPUT TAPE6,1016,IT
 1016 FORMAT(76H    PHI(K)=EXPECTED FUTURE COSTS RESULTING FROM THE BEGI
     XNNING OF TIME PERIODI3,23H WITH AN INVENTORY OF K)
   84 WRITE OUTPUT TAPE 6,1017,(MMK(I),PHI(I),I=LLJ,LLK)
 1017 FORMAT(119H1        K        PHI(K)        K        PHI(K)
     X K        PHI(K)        K        PHI(K)        K        PHI(K)/(
     C5(I10,F13.2,1H )))
   96 IF(LLK-K-1)86,87,87
   86 LLJ=LLK+1
      LLK=LLK+XMINOF(200,K+1-LLK)
      IF(IFLAG-1)84,84,87
   87 CONTINUE
  200 CONTINUE
      CALL EXIT
      END
      SUBROUTINE DATA
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      IF(JK)1,2,1
    1 READ INPUT TAPE 5,1001,K
```

```
      WRITE OUTPUT TAPE 6,1001,K
 1001 FORMAT(I5)
    2 IF(JCST)5,6,5
    5 READ INPUT TAPE 5,1002,CST
      WRITE OUTPUT TAPE 6,1002,CST
 1002 FORMAT(4F10.4)
    6 IF(JCIS)7,8,7
    7 READ INPUT TAPE 5,1002,CIS
      WRITE OUTPUT TAPE 6,1002,CIS
    8 IF(JCOT)9,10,9
    9 READ INPUT TAPE 5,1002,COT
      WRITE OUTPUT TAPE 6,1002,COT
   10 IF(JTHET)11,12,11
   11 READ INPUT TAPE5,1002,(TEMP1(I),I=1,4)
      WRITE OUTPUT TAPE6,1002,(TEMP1(I),I=1,4)
   12 IF(JRATE)13,14,13
   13 READ INPUT TAPE 5,1002,RATE
      WRITE OUTPUT TAPE 6,1002,RATE
   14 IF(JDIST)16,16,17
   17 IF(JDIST-3)18,18,16
   18 GO TO (20,30,40),JDIST
   20 READ INPUT TAPE 5,1003,XMEAN,DELTA
      WRITE OUTPUT TAPE 6,1003,XMEAN,DELTA
 1003 FORMAT(F10.4,F10.8)
      PRO(1)=1./EXPF(XMEAN)
      TEMP=PRO(1)
      DO 21 I=2,1000
      Z=I-1
      PRO(I)=PRO(I-1)*XMEAN/Z
      TEMP=TEMP+PRO(I)
      IF(TEMP+DELTA-1.)21,22,22
   21 CONTINUE
   22 IR=I
      DO 50 I=1,500
      IF(PRO(I)-DELTA)50,52,52
   50 CONTINUE
   52 IS(4001)=I
```

```
   16 RETURN
   30 READ INPUT TAPE 5,1003,XMEAN
      WRITE OUTPUT TAPE 6,1003,XMEAN
      XI=XMEAN*2.+1.
      IR=XI
      P=1./XI
      DO 31 I=1,IR
   31 PRO(I)=P
      RETURN
   40 READ INPUT TAPE 5,1001,IR
      READ INPUT TAPE 5,1004,(PRO(I),I=1,IR)
      WRITE OUTPUT TAPE 6,1004,(PRO(I),I=1,IR)
 1004 FORMAT(7F10.8)
      XMEAN=0.
      DO 41 I=1,IR
      Z=I-1
   41 XMEAN=XMEAN+PRO(I)*Z
      RETURN
      END
      SUBROUTINE THETA
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      LR1=1
      IUP=1
      IF(JK)90,10,90
   10 IF(C-TEMP1(3))30,20,30
   20 IF(D-TEMP1(4))30,60,30
   30 XKMN=-K-1
      IUP=IUP+K
      IF(A-TEMP1(1))50,40,50
   40 IF(B-TEMP1(2))50,150,50
   50 IUP=IUP+2*K
```

```
      GO TO 100
   60 IF(A-TEMP1(1))80,70,80
   70 IF(B-TEMP1(2))80,150,80
   80 XKMN=0
      LR1=2+K
      IUP=IUP+3*K
      GO TO 100
   90 XKMN=-K-1
      IUP=3*K+1
  100 A=TEMP1(1)
      B=TEMP1(2)
      C=TEMP1(3)
      D=TEMP1(4)
      DO140 LR=LR1,IUP
      XKMN=XKMN+1.
      IF(XKMN)5,6,7
    5 THET(LR)=C+D*XKMN
      GO TO 140
    6 THET(LR)=0.
      GO TO 140
    7 THET(LR)=A+B*XKMN
  140 CONTINUE
  150 RETURN
      END
      SUBROUTINE ASUBI
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      READ INPUT TAPE 5,501,JPA
  501 FORMAT(I5)
      READ INPUT TAPE 5,502,(PA(L),L=1,JPA)
      WRITE OUTPUT TAPE 6,502,(PA(L),L=1,JPA)
  502 FORMAT(7F10.8)
```

```
      SUMA=0.
      DO 10 L=1,JPA
      LL=JPA+1-L
      SUMA=SUMA+PA(LL)
      IF(SUMA+EPS-1.)10,20,20
   10 CONTINUE
      L=JPA
   20 MAXI=L
   21 ASUM=1.-SUMA
      IF(ASUM)25,26,26
   25 ASUM=0.
   26 CONTINUE
      RETURN
      END
      SUBROUTINE PSIONE
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      U=KK-1
      V=IRR-1
      PSI=(U-V)*CST
      CALL SAME
      RETURN
      END
      SUBROUTINE PSITWO
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
     CXMU(100),PA(100)
      DIMENSION TEMP1(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
     CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
     CSMALA,RATE,TEMP1
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      U=KK-1
```

```
 V=IRR-1
 PSI=(V-U)*COT
 CALL SAME
 RETURN
 END
 SUBROUTINE SAME
 DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
CXMU(100),PA(100)
 DIMENSION TEMP1(4)
 COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
CSMALA,RATE,TEMP1
 DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
 V=IRR-1
 IDUM=K+1+IRR-KK
 PSI=PSI+V*CIS+(1.-SMALA)*VMIN(IDUM)+SMALA*THET(IDUM)
 RETURN
 END
 END
```

FORTRAN program for Dynamic Markov Inventory Model in Chapter VI.

```
*     NAVC  CONTRACT NO. 155479-04-00
*     XEQ
C-------- +
      DIMENSION FNCOST(9,21)
      DIMENSION C2TAB(9,21)
      DIMENSION MIN(9,21)
      DIMENSION C1TAB(9,21)
      DIMENSION G1P11(21),G1P12(21),G2P21(21),G2P22(21),QL(9,21)
      DIMENSION G(2,26),P(2,2),D(6),IS1(21),IS2(21),IU1(21),IU2(21),
     1V01(21),V02(21),V11(21),V12(21),SE1C(21),SE2C(21),POLTAB(9,21),
     2COSTAB(9,21),PI(9)
      PI(1)=.1
      DO 10 I=2,9
      PI(I)=PI(I-1)+.1
   10 CONTINUE
      READ INPUT TAPE 5,1000,(FM1,FM2,TOLCON)
      IX=25
      EEXPM1=1./(EXPF(FM1))
      IF(EEXPM1-TOLCON)100,100,101
  101 G(1,1)=EEXPM1
      DO 102 I=1,IX
      X=I
      K=I+1
      QUOT=1.
      DO 103 J=1,I
      QUOT=QUOT*(FM1/X)
      X=X-1.
  103 CONTINUE
      G1=QUOT*EEXPM1
      IF(G1-TOLCON)104,104,105
  105 G(1,K)=G1
  102 CONTINUE
      GO TO 206
  104 KG1TOL=I
      GO TO 106
  100 KG1TOL=0
      GO TO 106
```

```
 206 KG1TOL=26
 106 EEXPM2=1./(EXPF(FM2))
     IF(EEXPM2-TOLCON)107,107,108
 108 G(2,1)=EEXPM2
     DO 109 I=1,IX
     X=I
     K=I+1
     QUOT=1.
     DO 110 J=1,I
     QUOT=QUOT*(FM2/X)
     X=X+1.
 110 CONTINUE
     G2=QUOT*EEXPM2
     IF(G2-TOLCON)111,111,112
 112 G(2,K)=G2
 109 CONTINUE
     GO TO 114
 111 KG2TOL=I
     GO TO 113
 107 KG2TOL=0
     GO TO 113
 114 KG2TOL=26
 113 READ INPUT TAPE 5,1001,(((P(I,J),J=1,2)I=1,2))
     READ INPUT TAPE 5,1002,((D(I),I=1,6),RHO,XPSI,DELTA,FLMBDA)
     LPSI=XPSI
C-------- -
     WRITE OUTPUT TAPE 6,4000,(G(1,N),N=1,KG1TOL)
     WRITE OUTPUT TAPE 6,4001,(G(2,N),N=1,KG2TOL)
4000 FORMAT(9H1 G(1,N)=,26F3.2)
4001 FORMAT(9H0 G(2,N)=,26F3.2)
C--------
C-------- +
C SETTING UP THE VALUE ITERATION TABLE
C-------- -
     L=(2*LPSI)+1
     DO 115 I=1,L
     IS1(I)=I
```

```
      IS2(I)=2
      IU1(I)=(LPSI+1)-I
      IU2(I)=(LPSI+1)-I
C EVALUATE SE1CU
C EVALUATION OF ESCU
      XU=IU1(I)
  340 IF(IU1(I))300,301,301
  301 IF(KG1TOL-IU1(I))302,303,303
  302 K1=KG1TOL
      GO TO 304
  303 K1=IU1(I)+1
  304 TERM1=0.
      DO 305 J1=1,K1
      XJ1=J1-1
      TERM1=TERM1+((XU-XJ1)*G(1,J1))
  305 CONTINUE
      TERM1=D(1)*TERM1
      IF(KG1TOL-IU1(I))306,306,307
  307 K2=KG1TOL-1
      TERM2=0.
      DO 308 J2=K1,K2
      XJ1=J2
      J3=J2+1
      TERM2=TERM2+((XJ1-XU)*G(1,J3))
  308 CONTINUE
      TERM2=D(2)*TERM2
      GO TO 309
  306 TERM2=0.
  309 IF(LPSI-IU1(I))310,310,311
  311 DIFF1=LPSI-IU1(I)
      TERM3=D(3)+(D(4)*DIFF1)
      GO TO 312
  310 TERM3=0.
  312 SE1C(I)=TERM1+(TERM2+TERM3)
      GO TO 313
  300 FU=IU1(I)
      TERM1=-D(2)*FU
```

```
      TERM2=0.
      IF(KG1TOL-1)314,314,315
  315 KG1=KG1TOL-1
      DO 316 J2=1,KG1
      XJ2=J2
      TERM2=TERM2+(XJ2*G(1,J2+1))
  316 CONTINUE
      TERM2=D(2)*TERM2
  314 IF(LPSI-IU1(I))317,317,318
  318 DIFF1=LPSI-IU1(I)
      TERM3=D(3)+(D(4)*DIFF1)
      GO TO 319
  317 TERM3=0.
  319 SE1C(I)=TERM1+(TERM2+TERM3)
C EVALUATE SE2CU
C EVALUATION OF ESCU
  313 XU=IU1(I)
      IF(IU2(I))500,501,501
  501 IF(KG2TOL-IU2(I))502,503,503
  502 K2=KG2TOL
      GO TO 504
  503 K2=IU2(I)+1
  504 TERM1=0.
      DO 505 J5=1,K2
      XJ2=J5-1
      TERM1=TERM1+((XU-XJ2)*G(2,J5))
  505 CONTINUE
      TERM1=D(1)*TERM1
      IF(KG2TOL-IU1(I))506,506,507
  507 K1=KG2TOL-1
      TERM2=0.
      DO 508 J6=K2,K1
      XJ2=J6
      J7=J6+1
      TERM2=TERM2+((XJ2-XU)*G(2,J7))
  508 CONTINUE
      TERM2=D(2)*TERM2
```

```
      GO TO 509
  506 TERM2=0.
  509 IF(LPSI-IU2(I))510,510,511
  511 DIFF1=LPSI-IU2(I)
      TERM3=D(3)+(D(4)*DIFF1)
      GO TO 512
  510 TERM3=0.
  512 SE2C(I)=TERM1+(TERM2+TERM3)
      GO TO 513
  500 FU=IU2(I)
      TERM1=-D(2)*FU
      TERM2=0.
      IF(KG2TOL-1)514,514,515
  515 KG2=KG2TOL-1
      DO 516 J2=1,KG2
      XJ2=J2
      TERM2=TERM2+(XJ2*G(2,J2+1))
  516 CONTINUE
      TERM2=D(2)*TERM2
  514 IF(LPSI-IU2(I))517,517,518
  518 DIFF1=LPSI-IU2(I)
      TERM3=D(3)+(D(4)*DIFF1)
      GO TO 519
  517 TERM3=0.
  519 SE2C(I)=TERM1+(TERM2+TERM3)
  513 CONTINUE
C EVALUATE G1P11
C EVALUATE G1*P11 AND G1*P12
  400 J3=(LPSI-IU1(I))+1
      IF(KG1TOL-J3)402,403,403
  402 G1P11(I)=0.
      GO TO 401
  403 G1P11(I)=G(1,J3)*P(1,1)
C EVALUATE G1P12
  401 J3=(LPSI-IU2(I))+1
      IF(KG1TOL-J3)405,406,406
  405 G1P12(I)=0.
```

```
      GO TO 404
  406 G1P12(I)=G(1,J3)*P(1,2)
C EVALUATE G2P21
C EVALUATE G2*P21 AND G2*P22
  404 IF(KG2TOL-J3)452,453,453
  452 G2P21(I)=0.
      GO TO 407
  453 G2P21(I)=G(2,J3)*P(2,1)
C EVALUATE G2P22
  407 IF(KG2TOL-J3)455,456,456
  455 G2P22(I)=0.
      GO TO 454
  456 G2P22(I)=G(2,J3)*P(2,2)
  454 CONTINUE
  115 CONTINUE
      KP=(2*LPSI)+1
      DO 20 J=1,KP
      V01(J)=300.
      V02(J)=300.
   20 CONTINUE
C     READ INPUT TAPE 9,1003,(V01(I),I=1,L)
C     READ INPUT TAPE 9,1004,(V02(I),I=1,L)
      IBOX=0
  124 DO 116 I=1,L
C EVALUATE V11(I)
C EVALUATION OF V11(I)
      TERM1=SE1C(I)
      TERM2=0.
      DO 600 J=1,L
      J1=L-J+1
      TERM2=TERM2+(G(1,J1)*P(1,1)*V01(J1))+(G(1,J1)*P(1,2)*V02(J1))
  600 CONTINUE
      V11(I)=TERM1+(RHO*TERM2)
C EVALUATE V12(I)
C EVALUATION OF V12(I)
      TERM1=SE2C(I)
      TERM2=0.
```

```
      DO 601 J=1,L
      J1=L-J+1
      TERM2=TERM2+(G(2,J1)*P(2,1)*V01(J1))+(G(2,J1)*P(2,2)*V02(J1))
  601 CONTINUE
      V12(I)=TERM1+(RHO*TERM2)
  116 CONTINUE
      DO 117 I=1,L
      CON=ABSF(V11(I)-V01(I))
      IF (CON-DELTA)118,118,119
  118 CONTINUE
  117 CONTINUE
      DO 120 I=1,L
      CON=ABSF(V12(I)-V02(I))
      IF (CON-DELTA)121,121,119
  121 CONTINUE
  120 CONTINUE
      GO TO 122
  119 DO 123 I=1,L
      V01(I)=V11(I)
      V02(I)=V12(I)
  123 CONTINUE
      IBOX=IBOX+1
C--------- +
      IF(SENSE SWITCH 1)170,124
  170 WRITE OUTPUT TAPE 6,2010,(IBOX)
 2010 FORMAT(56H1S  U  ESC(U)   G1*P1S   G2*P2S   VN(S,U)  ITERATION NO.
     1,I4)
      DO 171 I=1,L
      WRITE OUTPUT TAPE 6,2001,(IS1(I),IU1(I),SE1C(I),G1P11(I),G2P21(I),
     1V01(I))
  171 CONTINUE
      DO 172 I=1,L
      WRITE OUTPUT TAPE 6,2001,(IS2(I),IU2(I),SE2C(I),G1P12(I),G2P22(I),
     1V02(I))
  172 CONTINUE
C--------- +
      GO TO 124
```

```
  122 WRITE OUTPUT TAPE 6,2000
      DO 125 I=1,L
      WRITE OUTPUT TAPE 6,2001,(IS1(I),IU1(I),SE1C(I),G1P11(I),G2P21(I),
     1V01(I))
  125 CONTINUE
      DO 126 I=1,L
      WRITE OUTPUT TAPE 6,2001,(IS2(I),IU2(I),SE2C(I),G1P12(I),G2P22(I),
     1V02(I))
  126 CONTINUE
C -----------
C MINIMIZATION OF FIRST PROBLEM
C -----------
      L=(2*LPSI)+1
      PIVOT=(PI(1)*V11(1))+((1.-PI(1))*V12(1))
      DO 700 I=1,9
      DO 701 J=1,L
      POLTAB(I,J)=LPSI
      COSTAB(I,J)=(PI(I)*V11(J))+((1.-PI(I))*V12(J))
      COSTAB(I,J)=COSTAB(I,J)-PIVOT
      IF(IU1(J))850,851,851
  851 U=IU1(J)
      SUM7=0.
      LU1=IU1(J)+1
      DO 852 K=1,LU1
      X=K-1
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)
      HPIX=A1+A2
      SUM7=SUM7+((U-X)*HPIX)
  852 CONTINUE
      SUM7=D(1)*SUM7
      LU1=LU1+1
      SUM8=0.
      DO 853 K=LU1,L
      X=K-1
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)
```

```
      HPIX=A1+A2
      SUM8=SUM8+((X-U)*HPIX)
  853 CONTINUE
      SUM8=D(2)*SUM8
      GO TO 854
  850 U=IU1(J)
      SUM7=(-D(2)*U)
      SUM8=0.
      DO 855 K=1,L
      X=K-1
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)
      HPIX=A1+A2
      SUM8=SUM8+(X*HPIX)
  855 CONTINUE
      SUM8=D(2)*SUM8
  854 C2TAB(I,J)=SUM7+SUM8
  701 CONTINUE
  700 CONTINUE
      WRITE OUTPUT TAPE 6,3002
 3002 FORMAT(115H1 COSTAB      PI=.1        PI=.2        PI=.3        PI=.4
     1       PI=.5        PI=.6        PI=.7        PI=.8        PI=.9)
      DO 3001 J=1,L
      WRITE OUTPUT TAPE 6,3000,(IU1(J),(COSTAB(I,J),I=1,9))
 3000 FORMAT(5H0  U=,I5,4X,8(F6.2,6X),F6.2)
 3001 CONTINUE
      NU=(2*LPSI)+1
      NPI=9
      JBOX=1
  799 DO 800 I=1,NPI
      DO 801 J=1,NU
      LU=0
  804 CONTINUE
      SQ=POLTAB(I,J)
      ISQ=SQ
      IF(ISQ-LPSI)1200,1201,1200
 1200 IF(ISQ-LU)1202,1203,1202
```

```
1202 KKK=1
     GO TO 999
1301 QSMQ2=CQL
     ISQ=ISQ-1
     KKK=2
     GO TO 999
1302 QSMQ1=CQL
     ISQ=ISQ+2
     KKK=3
     GO TO 999
1303 QSMQ3=CQL
     ISQ=ISQ-1
     IF(QSMQ2-QSMQ1)1205,1205,1204
1204 IF(ISQ-1-LU)1206,1206,1207
1207 ISQ=ISQ-1
     GO TO 1202
1206 QL(I,J)=QSMQ1
     MIN(I,J)=ISQ-1
     GO TO 1210
1205 IF(QSMQ2-QSMQ3)1208,1208,1209
1208 QL(I,J)=QSMQ2
     MIN(I,J)=ISQ
     GO TO 1210
1209 IF(ISQ+1-LPSI)1211,1212,1212
1212 QL(I,J)=QSMQ3
     MIN(I,J)=ISQ+1
     GO TO 1210
1211 ISQ=ISQ+1
     GO TO 1202
1203 KKK=4
     GO TO 999
1304 QSMQ2=CQL
     ISQ=ISQ+1
     KKK=8
     GO TO 999
1308 QSMQ3=CQL
     ISQ=ISQ-1
```

```
      IF(QSMQ2-QSMQ3)1213,1213,1214
 1213 QL(I,J)=QSMQ2
      MIN(I,J)=ISQ
      GO TO 1210
 1214 IF(ISQ+1-LPSI)1215,1216,1216
 1216 QL(I,J)=QSMQ3
      MIN(I,J)=ISQ+1
      GO TO 1210
 1215 ISQ=ISQ+1
      GO TO 1203
 1201 IF(LPSI-LU)1217,1217,1218
 1217 KKK=5
      GO TO 999
 1305 QL(I,J)=CQL
      MIN(I,J)=ISQ
      GO TO 1210
 1218 KKK=6
      GO TO 999
 1306 QSMQ2=CQL
      ISQ=ISQ-1
      KKK=7
      GO TO 999
 1307 QSMQ1=CQL
      ISQ=ISQ+1
      IF(QSMQ2-QSMQ1)1217,1217,1219
 1219 IF(ISQ-1-LU)1220,1220,1221
 1220 QL(I,J)=QSMQ1
      MIN(I,J)=ISQ-1
      GO TO 1210
 1221 ISQ=ISQ-1
      GO TO 1218
 1210 CONTINUE
C EVALUATE C1(U,PI) AND STORE IN C1TAB(I,J)
      C1TAB(I,J)=C2TAB(I,J)+QL(I,J)
      PIVOT=C2TAB(1,1)+QL(1,1)
      POLTAB(I,J)=MIN(I,J)
      C1TAB(I,J)=C1TAB(I,J)-PIVOT
```

```
 801 CONTINUE
 800 CONTINUE
     DO 820 I=1,NPI
     DO 821 J=1,NU
     DIFF=ABSF(C1TAB(I,J)-COSTAB(I,J))
     IF(DIFF-FLMBDA)822,822,823
 822 CONTINUE
 821 CONTINUE
 820 CONTINUE
     GO TO 824
 823 DO 825 I=1,NPI
     DO 826 J=1,NU
     COSTAB(I,J)=C1TAB(I,J)
 826 CONTINUE
 825 CONTINUE
     IF(SENSE SWITCH 4)3051,3052
3051 WRITE OUTPUT TAPE 6,3002
     DO 3050 J=1,NU
     WRITE OUTPUT TAPE 6,3000,(IU1(J),(COSTAB(I,J),I=1,9))
3050 CONTINUE
     WRITE OUTPUT TAPE 6,2502
     DO 3060 J=1,NU
     WRITE OUTPUT TAPE 6,3000,(IU1(J),(POLTAB(I,J),I=1,9))
3060 CONTINUE
     WRITE OUTPUT TAPE 6,3099,(JBOX)
3099 FORMAT(14H0ITERATION NO.,I4)
3052 CONTINUE
     IF(SENSE SWITCH 3)5010,5009
5009 JBOX=JBOX+1
     GO TO 799
 824 CONTINUE
     SCON=PIVOT/(1.-RHO)
     DO 888 I=1,NPI
     DO 889 J=1,NU
     FNCOST(I,J)=C1TAB(I,J)+SCON
 889 CONTINUE
 888 CONTINUE
```

```
      WRITE OUTPUT TAPE 6,2500
 2500 FORMAT(115H1 COSTAB      PI=.1        PI=.2        PI=.3        PI=.4
     1       PI=.5        PI=.6        PI=.7        PI=.8        PI=.9)
      DO 875 J=1,NU
      WRITE OUTPUT TAPE 6,2501,(IU1(J),(FNCOST(I,J),I=1,9))
 2501 FORMAT(5H0  U=,I5,4X,8(F6.2,6X),F6.2)
  875 CONTINUE
      WRITE OUTPUT TAPE 6,2502
 2502 FORMAT(115H1 POLTAB      PI=.1        PI=.2        PI=.3        PI=.4
     1       PI=.5        PI=.6        PI=.7        PI=.8        PI=.9)
      DO 876 J=1,NU
      WRITE OUTPUT TAPE 6,2501,(IU1(J),(POLTAB(I,J),I=1,9))
  876 CONTINUE
      WRITE OUTPUT TAPE 6,3029,(JBOX)
 3029 FORMAT(25H0 THE COST TABLE REQUIRED,I3,11H ITERATIONS)
 5010 CONTINUE
      CALL DUMP
  999 FISQ=ISQ
      L=ISQ
      SUM1=0.
      DO 806 K=1,NU
      X=K-1
C EVALUATE C(L-X,PI'(X)).HPI(X) AND STORE IN SUMA
      KX=K
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)
      B1=P(1,1)*A1
      B2=P(2,1)*A2
      HPIX=A1+A2
      PRIME=(B1+B2)/HPIX
      K1=INTF(10.*PRIME)
      K2=K1+1
      IF(L-KX+1+LPSI)950,951,951
  951 K3=LPSI+KX-L
      T1=COSTAB(K2,K3)-COSTAB(K1,K3)
      T2=(PRIME-PI(K1))/.1
      SUMA=T1*T2+COSTAB(K1,K3)
```

```
      GO TO 953
  950 S1=COSTAB(1,K1)-((COSTAB(NU,K1)-COSTAB(1,K1))*(FISQ-X))
      S2=COSTAB(1,K2)-((COSTAB(NU,K2)-COSTAB(1,K2))*(FISQ-X))
      T1=S2-S1
      T2=(PRIME-PI(K1))/.1
      SUMA=T1*T2+S1
  953 SUM1=SUM1+(SUMA*HPIX)
  806 CONTINUE
      IF(L-IU1(J))807,808,807
  808 SUM2=0.
      GO TO 809
  807 DIFF=L-IU1(J)
      SUM2=D(3)+(D(4)*DIFF)
  809 CQL=(RHO*SUM1)+SUM2
      GO TO (1301,1302,1303,1304,1305,1306,1307,1308),KKK
 1000 FORMAT(6X,F9.5,1X,F9.5,E12.4)
 1001 FORMAT(5X,4(1X,F9.5))
 1002 FORMAT(7X,8(F5.2),1X,2F5.3)
 2000 FORMAT(41H1S  U  ESC(U)   G1*P1S   G2*P2S   VT(S,U))
 2001 FORMAT(1H0,I1,1X,I2,2X,F6.2,3X,F6.2,3X,F6.2,3X,F7.2)
C THIS IS THE END OF PROGRAM
      END
```

Appendix C

Discussion of ASO Procedures

A. Demand ("Requirement") Prediction

Two systems of demand prediction are in use at ASO: The Program Usage Replenishment System (PURS) and the Replenishment Demand Issue System (RDIS). Under both systems total demand is estimated for a time period equal to lead time (production lead plus activity lead plus administrative lead) plus a six month "safety level" time plus a six month "order cycle". Safety level and order cycle do not vary from item to item, although lead time does somewhat. The first two numbers are established by fiat and are independent of costs associated with items (holding, purchase, stockout, etc.); so, since the lead time cannot be manipulated, it is clear that there is no room for optimization under either PURS or RDIS.

Under PURS, past usage records are kept in terms of items consumed by aircraft maintenance cycle (240 hours for all aircraft except jets, for which it is 60 hours). These records are divided into a maintenance rate (items consumed per flying hour) and an overhaul requirement (items consumed in overhaul at the end of the cycle). These records are gathered from user activities in the field.

From CNO and BUWEPS come estimates of expected flying hours and overhaul schedules during the following six months and for the ninth month from payment. A technical file is maintained which provides information on item applications. There appears to be a very slight relationship between active lead time and the figure used in these calculations.

PURS then uses the usage records, expected future flying hours, and the technical file to produce a requirement for each item.

Two major weaknesses are inherent in PURS. The first is the arbitrariness of the demand period considered and the second is the unreliability of the data. Usage records are poorly kept at activities and, further, no significant correlations have been found either between predicted flying hours and actual flying hours or between actual flying hours and demand.

Under the second demand prediction system, RDIS, the assumption is made that future demand will be the same as past demand. This system has the advantage of simplicity, and it does not rely on as much unreliable data as does PURS. However, its demand estimates are no better than those generated by PURS, and it has less scientific appeal.

B. Requirement Adjustment for Repairables

For repairable items, predicted demand is reduced by predicted returns of items which have successfully passed the maintenance cycle. The maintenance cycle is defined as six months for most items, though only three for some high value articles. Time spent by an item in the cycle is referred to as "turnaround time". The cycle consists of three states: a) the item is removed from an aircraft and screened for repairability. If it looks satisfactory, then b) it goes to the maintenance depot where it is screened again. If it is still alright, it is entered in the stock records and c) goes to the overhaul shop.

There it gets the final stamp of approval, is repaired, and is put on the shelf.

The "recovery rate" is defined as the fraction of items removed from aircraft which return from the maintenance cycle. The "RB recovery rate" is the fraction of those which are not discarded at removal which return from stages b) and c) of the cycle.

The order requirement for repairables (O) is calculated as gross expected demand (D) minus expected item removals (R_i) times recovery rate (r) minus damaged equipment (d) reported on hand times RB (RB) recovery rate minus amount (H) on hand and on order.

$$O = D - R_i \times r - d \times RB - H$$

The time period considered for demand is as defined for PURS and RDIS, the sum of lead time, safety level and order cycle. The removal period begins a turn-around time earlier and ends a turnaround time plus a safety level earlier.

C. <u>Life of Type Items</u>

At one time the Navy estimated its requirement for some items by "calculating" total demand overall time and reducing this by estimated recoveries. This method was usually used for high recovery rate items. At present, however, they buy only eighteen months worth of such items. This generally amounts to the same thing, when safety factors have been taken into account.

ASO has a formula which it uses to decide whether to dispose of some life-of-type aircraft parts. The formula produces the expected number of maintenance cycles in the remaining life of the aircraft to which the part is applied; this number multiplied by the appropriate usage rate produces a desired number of parts on hand. If this is less than the actual number on hand, then ASO will dispose. The formula is:

$$\text{Number of maintenance cycles in life of item} = \frac{Nh}{240}\left[\int_{o}^{x} a^{t}dt - je\int_{o}^{x-kb} a^{t}dt\right]$$

where N is number of aircraft, h is average monthly flying hours per aircraft, a is aircraft survival rate, e is system recovery fraction, x is number of months of remaining life averaged over all aircraft calculated from service tours remaining, k is number of months turnaround time plus safety level, g is fraction of aircraft operating, and j is a safety factor which ASO has introduced as a hedge against the possible unreliability of this formula.

D. Economic Order Quantity

A procedure for calculating an "economic order quantity" (EOQ) is applied to some consumable items. It produces an operating level of an item to which lead time, etc., are then added. The EOQ is required to lie between the operating level calculated on the basis of estimated demand over the time period mentioned above and the life of type estimate of the item. It is designed to correct against uneconomically small orders, but cannot prevent the more usual uneconomically large orders. The EOQ procedure also provides a scientific method for computing a safety level as follows:

a) One computes $\sigma_{DL} = \sqrt{\bar{D}^2\bar{L} + \bar{L}\,\bar{D}}$ where $\bar{D}$ is expected demand and $\bar{L}$ is expected lead time. The first $\bar{L}$ and the second $\bar{D}$ in the equation are approximations to standard deviations of demand and lead time.

b) One computes $R = (Q \times I \times C)/(T \times H \times E)$ where Q is the economic order quantity, I is a discount factor which includes interest, obsolescence, and holding cost, C is the unit price, T is estimated annual demand, H is the shortage cost, and E is "military essentiality."

c) One uses I-R and a table of normal variates to find K, the number of multiples of standard deviation.

d) One declares that the safety level is K times σ_{DL}, This is assumed to be in units of quantity, dimensional analysis to the contrary notwithstanding.

E. Critique of These Procedures

A major handicap in the Navy's thinking about supply procedures lies in the depiction of expected demand estimates, based on suspect data and an arbitrary time frame, as "requirements." Once a course of action having the repercussions of this one has been called a requirement to be met at all cost, then no quantity of peripheral models and techniques will introduce a significant increase in supply efficiency. If the Navy has any desire to reduce the huge cost of its supply operation it must have a system which considers simultaneously demand, purchase costs, holding costs, and outage costs, and which neither equates quantities with times nor substitutes mean demands for distributions of demands.

If the concept of "safety-level" had any operational meaning, then it clearly should vary from item to item. In fact, however, it does not have meaning. For a given state of the inventory system, there exists a best quantity of each item to order. Other things remaining constant, this best quantity will vary with the production set-up cost of the item. If there were such a thing as a safety level, then, it would have to be something which is a function of set-up cost. This is clearly a contradiction in terms.

F. <u>The Data System</u>

There are two major weaknesses in the ASO data collection and maintenance procedures. The first is the unreliability of what is collected and the second is the choice of what ought to be collected. The unreliable data problem is one which ASO can hardly solve by itself, but which must be faced by the Navy as a whole; however, something can be done about deciding what data would be useful to have.

Certainly summaries of the obsolescence experience of items should be kept--e.g., for each category of item, the average lifespan of its members. Similarly, summaries of demand history should be maintained. Stockout reports should be input to the data collection system. This sort of information will be required by any effective inventory control model, and, together with such a model, could enable a truly efficient operation to be carried on.

REFERENCES

[1] Allen, S.G., and T.R. Broida; Stock Control Policy, SRI Project No. LU-2109, Stanford Research Institute, Menlo Park, California, 1958

[2] Barankin, E.W., and J. Denny; "Examination of an Inventory Model Incorporating Probabilities of Obsolescence," A Working Paper, Statistical Laboratory, University of California, Berkeley, 1961

[3] Brown, G.W.; This Report
C-E-I-R, Inc., Los Angeles Center

[4] Ford, L.R., Jr.; This Report
C-E-I-R, Inc., Los Angeles Center

[5] Fukuda, Yoichiro; Optimal Disposal Policies,
Planning Research Corporation, Los Angeles, California, 1960

[6] Grassi, R.C., and A.J. Gradwohl; "Obsolescence and Economic Lot Size, "Journal of Industrial Engineering, Vol. X, No. 5, 1959

[7] Hadley, G., and T.M. Whitin; "An Optimal Final Inventory Model," Management Science, Vol. 7, No. 2, January 1961.

[8] Hadley, G.; "Generalization of the Optimal Final Inventory Model," Unpublished paper, University of Chicago, 1961

[9] McGlothlin, W.H. and R. Radner; "The Use of Bayesian Techniques for Predicting Spare-Parts Demand", RM 2536, The RAND Corporation, Santa Monica, March 1, 1960

[10] McGlothlin, W.H., and E.E. Bean: "Application of Bayesian Techniques to Spare-Parts Demand Prediction", RM 2701, The RAND Corporation, Santa Monica, January 3, 1961

[11] Mohan, C., and R.C. Garg; "Decision on Retention of Excess Stock Following a Normal Probability Law of Obsolescence and Deterioration," Naval Research Logistics Quarterly, Vol. 8, No. 3, September 1961

[12] Simpson, J.R.; "A Formula for Decision on Retention or Disposal of Excess Stock, "Naval Research Logistics Quarterly, Vol. 2, No. 3, Sept. 1955

[13] Whitin, T.; The Theory of Inventory Management, Princeton University Press, 1957